The Composition of Final Uses, R&D, and Growth

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Abstract

Almost all (non-defense) R&D in the US is undertaken by industries which directly account for most of the (non-construction) capital formation, and close to half of all the final uses of output. While most endogenous growth models cannot account for a direct association between R&D and the production of final goods, the current paper links the composition of final uses, R&D and growth. Some producers of final goods in the model economy choose to invest in enhancing their productivity, while others choose to use the commonly known state-of-the-art technology. The resulting steady-state equilibrium is consistant, under certain parameter values, with some basic features of the US economy, including the above breakdown of R&D and final uses. It is also shown that while growth and saving both increase as a result of a permanent improvement in the R&D technology, in the short-run saving and growth may be negatively correlated. Furthermore, capital goods prices and growth rates may exhibit negative correlation, in accordance with the data.

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1. Introduction

The manufacturing sector in the US accounted for 75% of all industrial non-defense $R\&D$ expenditures in the US in 1992. The same sector accounted for 89% of fixed investment (excluding construction), and also for 40% of personal consumption (excluding housing services). The sum of fixed investment and personal consumption associated with the manufacturing sector constituted 46% of these final uses (again, excluding the housing sector).[1] While $R\&D$ activities seem to be most closely related to the production of capital goods, these data clearly indicate that many of the consumption goods are directly connected to these activities as well.

The relationship between the composition of final uses, $R\&D$ activities and economic growth has not attracted much attention. Existing endogenous growth models emphasize the role of $R\&D$ through its impact on productivity at the intermediate goods level, and ignore the direct and significant association between $R\&D$ activities, consumption, and other final uses. Typically, the model economies analyzed in the literature contain a competitive aggregate final good sector, an intermediate good sector and a knowledge creating sector. Knowledge is used to improve the intermediate goods which in turn are purchased by the producers of the final good as inputs (e.g. Romer (1990)). In this structure $R\&D$ activities are only indirectly related to the final good sector. In particular, the incentives to engage in $R\&D$ have little to do with the demand for the consumption

[1] This data is based on Table C of the Benchmark Input-output Accounts of the United States, 1992, and on Table B17 of the NSF *National Patterns of R&D Resources: 1998*. The aggregation is based on the sic codes reported in Table B17 above, and on the association between the input-output commodity numbers and their corresponding sic codes listed in Appendix A of the Benchmark Input-output Accounts of the United States, 1992.

good. Moreover, taken literally, such models predict that no $R\&D$ activity takes place in the final goods sector, especially by producers of consumption goods.[2] Therefore, such models are ill suited to address the phenomena listed above, and cannot be applied in explorations of the connection between the structure of final demand, $R\&D$ and economic growth.[3]

This paper proposes a model designed to generate implications which roughly mimic the sectoral structure of the final uses in the US, both qualitatively and quantitatively. The model is based on a tradeoff, faced by *each* individual producer of final goods, in deciding on how to use its resources. Specifically, it is up to the producer to decide whether or not to invest some of its resources in $R\&D$ rather than allocate all resources to the production of goods. $R\&D$ activities involve a quasi-fixed cost, so that not all producers opt to invest in $R\&D$ and become innovators. In equilibrium, the parameters characterizing the costs and benefits of $R\&D$ relative to those of producing final goods determine the size of the knowledge-based sector of producers which choose to engage in $R\&D$. Overall growth in the economy is due solely to knowledge creation, and therefore the same factors that affect the size of the knowledge sector, including the determinants of

[2] Quality-ladder models also dissociate the $R\&D$ activities from the production of of final goods, since the former are undertaken only by firms that seek to enter the product market, and are stopped once they have succeeded in their quest, (see, for instance, Grossman and Helpman (1991), and Aghion and Howitt (1992)).

[3] A recent OECD study concluded that a broad definition of the so-called "knowledge economy", which includes creators, distributors and users of high technology, accounts for over 50% of business value added in most western countries (Benchmarking Knowledge-based Economies, OECD Science, Technology and Industry, Scoreboard 1999.) Notice that a value added account cannot distinguish between the role of $R\&D$ in the intermediate goods and final goods markets.

the demand of final goods, also affect the economy's growth rate.

On the demand side, our model-economy is quite standard. There are infinitely lived consumers who maximize the expected present value of the utility stream they obtain from consuming various goods which are imperfectly substitutable. These consumers transfer income from period to period by purchasing a capital good which they rent out in the following period. In addition, they obtain income from a unit of labor with which they are endowed every period and from the profits generated by the producers in the economy.

Producers use capital and labor in a constant returns to scale technology to generate output. Every producer can costlessly divide this output into a capital good, which is indistinguishable from other capital goods, and a specific consumption good, over which the producer has some monopoly power.

The production process of every producer is affected by an individually chosen technology factor. Specifically, at every period each producer may choose to adopt the latest technology which is freely available economy-wide. Alternatively, a producer may choose to allocate some productive resources to $R\&D$ in order to create a better technology. As it turns out, asymmetric $R\&D$ participation choices in equilibrium drive the interesting results in this paper.

The freely available default technology in the economy sets a technological *threshold* which acts like a quasi-fixed element in the cost of $R\&D$.[4] Consequently, producers choose to engage in $R\&D$ as long as the diversion of resources from pro-

[4] Technically we assume that in the process of creating a new technology, the available one has to be reinvented. It is through this feature that we obtain a non-convexity in the $R\&D$ cost which distinguishes our approach from most of the endogenous growth literatue. Usually, current $R\&D$ is assumed to improve upon the level of "knowledge" by *incrementally* adding to it, rather than by *radically* replacing it, and there is no minimal requirement on $R\&D$ investment.

duction activities to $R\&D$ is rewarded by a sufficient increase in profits. With free entry into $R\&D$ activities we obtain that each period producers self-select to belong to one of two sectors: a "knowledge-based" sector of those who engage in $R\&D$, and a sector of those who produce their final goods using the default technology. The capital good turns out to be produced in equilibrium only by producers who belong to the "knowledge-based" sector. Moreover, the consumption goods produced by the "knowledge based" sector turn out to be cheaper than those produced by the other producers. The asymmetry in $R\&D$ participation occurs in equilibrium because the relative size of the two sectors affects the relative price of their consumption goods, and hence the profitability of $R\&D$. These asymmetries in $R\&D$-related activities in the presence of quasi-fixed costs provide new insights not previously present in works on $R\&D$ and growth.[5]

The improvements created by the "knowledge-based" sector are assumed to diffuse over time to the remainder of the economy, thereby improving the technology all producers can costlessly adopt. This implies, by the threshold effect on $R\&D$, that the amount of resources required to improve upon the freely available technology is growing over time.[6] Absent any exogenous growth in resources, economic growth can be sustained only if the technological improvements generate

[5] Previous papers incorporated quasi-fixed $R\&D$ costs but assumed symmetric $R\&D$ participation decisions. The consequence is a two-phase equilibrium path: Any $R\&D$ phase is followed by a phase in which investment is redirected to capital and not to $R\&D$. A new $R\&D$ phase starts once the economy has accumulated sufficient resources to warrant the quasi-fixed $R\&D$ costs. See Bental and Peled (1996) and Matsuyama (1999).

[6] The assumption that $R\&D$ costs increase as the economy becomes technologically more advanced, sometimes refered to as "increasing complexity", is common to many $R\&D$-driven growth models, (see Bental and Peled (1996), Matsuyama (1999), and Howitt (1999)).

sufficiently high output growth to support the necessary increasing $R\&D$ efforts.[7]

A dynamic equilibrium path of this economy consists, among other things, of its output growth and saving rate. We examine the effect of some changes in the underlying parameters on the equilibrium path, and compare some of its properties to those of alternative endogenous growth models and to the data. In particular, we show that a permanent improvement in the $R\&D$ technology increases, as expected, both output growth and the saving rate in the steady-state. However, simulations of a linearized version of the model around its steady-state reveal that shocks to the $R\&D$ technology may generate negative "short run" correlations between the saving rate and growth. These results are consistent with some observations that have been used as evidence against standard versions of endogenous growth models, (see Jones (1995b)).

We describe the interdependence of endogenous $R\&D$ participation decisions and growth within a general version of the model in the next two sections. The equilibrium of a parametrically specified version of the model is presented in section 4. In section 5 we compute and compare steady-states, focusing on the impact of changes in preferences and $R\&D$ technology. In section 6 we linearize the model to compute relationships among stochastic deviations of key variables from their non-stochastic steady state levels. We compare some of the results to analogous correlations in data from the G-7 countries. Further implication of the model are discussed in section 7.

[7] Some endogenous growth models, in which $R\&D$ is subject to decreasing returns, require exogenous growth in an input which is used in the $R\&D$ process, (e.g. Jones (1995), Kortum (1997)). Our setting supports sustained growth without such assumptions.

2. The Environment

Time is discrete, indexed by $t = 1, 2, ...$, and the economy is populated by a continuum of infinitely lived, identical consumers of unitary measure. Each consumer is endowed with 1 unit of labor per period which is supplied inelastically.[8]

The periodic consumer preferences are defined over a continuum of non-storable consumption goods on the unit interval, indexed by $i \in [0, 1]$. The different consumption goods are combined to generate composite consumption according to:

$$C_t = \Phi\left(c_t(\cdot)\right) \tag{2.1}$$

where Φ is a concave functional, and $c_t(i)$ is the amount consumed of good $i \in [0, 1]$. The consumer enjoys a periodic utility flow of $u(C_t)$, generated by a strictly increasing and concave function $u(\cdot)$, and a lifetime discounted utility of:

$$\sum_{t=1}^{\infty} \beta^t u(C_t). \tag{2.2}$$

There is a continuum of producers of unitary measure.[9] All producers employ capital and labor to produce output but may differ in the technologies they employ.

The output of producer i, $y_t(i)$, can be costlessly and irreversibly divided into two distinct versions: a producer-specific consumption good, $c_t(i)$, and a capital good, $k_{t+1}(i)$,

$$y_t(i) = c_t(i) + k_{t+1}(i), \quad \forall i \in [0, 1]. \tag{2.3}$$

[8] We fix the size of the population to underscore the fact that population growth is not needed to support sustained growth.

[9] The assumption that the measure of producers is fixed implies a fixed employment per producer in a symmetric equilibrium. This assumption reflects an implicit limit on the size of the labor force that can be effectively managed by each producer. Matuyama (1992) adopts a similar assumption to avoid population scale effects. See also Lucas (1993).

The capital goods of all producers are perfect substitutes. Accordingly, at the end of each period the newly produced capital goods of all producers are aggregated to form the economy's investment. The economy's capital stock at period $t+1$ is given by:

$$K_{t+1} = \int_0^1 k_{t+1}(i)di + (1-\delta)K_t, \tag{2.4}$$

where δ is the depreciation rate.

The amount of output produced by any producer, (both the consumption and capital versions), depends on a technology level, (A), capital input, (k), and labor input, (l), according to $F(A, k, l)$. We assume that for any A, $F(A, \cdot, \cdot)$ is concave, with constant returns to scale in (k, l). Thus, output of producer i at time t is given by:

$$y_t(i) = F\left[A_t(i), k_t^y(i), l_t^y(i)\right], \tag{2.5}$$

where $k_t^y(i)$ and $l_t^y(i)$ denote capital and labor employed to produce good i, and $A_t(i)$ is the technology in use by producer i at time t.

The technology with which each good is produced may be chosen by each producer, as follows. At each period t every producer may adopt at no cost a default technology, A_{t-1}, which reflects past technological improvements in the economy. However, instead of using the default A_{t-1}, each producer may independently opt to develop a new production technology, $\psi_t(i)$, by his own $R\&D$ efforts so that:

$$A_t(i) = \max\left\{A_{t-1}, \psi_t(i)\right\} \tag{2.6}$$

Developing one's own technology requires capital and labor resources which could otherwise be used for production of goods with the default technology, but the resulting (superior) technology is immediately applicable. It is important to emphasize that a new technology developed by any producer is exclusively available to that producer *only* during the current period. In subsequent periods, all the technologies that were in use, both new and old, are combined to create the economy-wide, freely available default technology, which is explained below.[10]

The technology generated by individual *R&D* activities is also affected by an economy-wide random shock, B, that represents exogenous, economy-wide, *R&D* conditions.[11] This shock is assumed to be generated by the following stochastic process

$$B_t = B_0 + rB_{t-1} + e_t, \; -1 < r < 1, \tag{2.7}$$

:where B_0 is a given constant and e_t is a zero-mean i.i.d. process.

Given B_t, the technology generated by the *R&D* activity of the i^{th} producer during that period is:

$$\psi_t(i) = G\left[k_t^R(i), l_t^R(i), B_t\right] \tag{2.8}$$

where $k_t^R(i)$ and $l_t^R(i)$ are the capital and labor inputs allocated to *R&D* and $G(\cdot, \cdot, B)$ has constant returns to scale in (k, l) for any B. Obviously, given the freely available default technology A_{t-1}, allocating resources to create a new technology for good i is potentially beneficial according to (2.6) only if $\psi(i) > A_{t-1}$.

The new technological default for period $t+1$, A_t, is assumed to depend on all the technologies employed in the production of goods at period t. It also depends

[10] The limited duration of proprietary rights over innovations before they affect the commonly known and freely available default technology should be viewed as an enviromental feature reflecting knowledge spillover or diffusion rates. See Bental and Peled (1996) and Matsuyama (1999), among others, for similar assumptions.

[11] We have in mind the discovery of some general purpose technologies or scientific principles which affect productivity of all *R&D* efforts in the economy.

on the measure of producers who engage in $R\&D$ in period t, denoted by λ_t.[12] Accordingly:

$$A_t = A\left[\psi_t(\cdot), A_{t-1}, \lambda_t\right]. \tag{2.9}$$

Finally, the resource allocation in this economy at any period t, given capital stock K_t and labor force size normalized to unity, is constrained by:

$$\begin{aligned} K_t &= \int_0^1 \left[k_t^y(i) + k_t^R(i)\right] di \\ 1 &= \int_0^1 \left[l_t^y(i) + l_t^R(i)\right] di. \end{aligned} \tag{2.10}$$

3. A Decentralized Equilibrium

3.1. Consumers

Each period t, consumers choose a mix of saving and consumption goods to maximize their expected utility. Taking prices, profits, wages and rates of return on saving as given, their choice is restricted by the budget and capital accumulation constraints:

$$\int_0^1 p_t(i)c_t(i)di + \int_0^1 p_{kt}(i)k_{t+1}(i)di = w_t + r_tK_t + \int_0^1 \pi_t(i)di \tag{3.1}$$

$$K_{t+1} = (1-\delta)K_t + \int_0^1 k_{t+1}(i)di \tag{3.2}$$

[12] Qualitatively, the model does not depend on the existence of this externality. Howevetr, we use the externality below to generate "reasoanble" growth rates. We also believe that such externalities exist in reality, reflecting on-the-job training of researchers whose knowledge spreads through the economy by job switching. When more producers engage in R&D, researchers are likely to be more mobile and diffusion of knowledge becomes easier.

where $p_t(i)$ and $p_{kt}(i)$ are the prices of the i^{th} consumption and capital goods, respectively, w_t is the wage rate, r_t is the rental rate on capital, and $\pi_t(i)$ denotes time t profits of producer i.

Since all capital goods are perfect substitutes they must be equally priced in equilibrium. We normalize their common price to unity,

$$p_{kt}(i) = 1,\ i \in [0, 1]\,,\ \forall t. \tag{3.3}$$

All other time t consumption and factor prices are expressed in terms of this unitary price of the saving good.

Let the state of the economy at time t be given by $Z_t = (B_t, K_t, A_{t-1})$, which includes, (but is not restricted to), the current realization of the $R\&D$-productivity shock, the stock of capital at the beginning of the period, and the available default technology. We are interested in an equilibrium with time-invariant functions relating prices and profits at time t to the state of the economy at t:

$$p_t(i) = p(i, Z_t), \quad \forall i \tag{3.4}$$

$$w_t = w(Z_t), \tag{3.5}$$

$$r_t = r(Z_t) \tag{3.6}$$

$$\pi_t(i) = \pi(i, Z_t), \quad \forall i \tag{3.7}$$

Let q_t be the consumer's current income at t in terms of the saving good. The corresponding Bellman equation for the consumer's problem is:

$$W(q_t) = \max_{\{c_t(i), k_{t+1}(i), i \in [0,1]\}} \{u(C_t) + E[W(q_{t+1}(Z_{t+1})) \mid Z_t]\} \tag{3.8}$$

subject to:

$$C_t = \Phi(c_t(\cdot)) \tag{3.9}$$

$$q_t = \int_0^1 p_t(i)c_t(i)di + \int_0^1 k_{t+1}(i)di \tag{3.10}$$

$$q_{t+1}(Z_{t+1}) = w(Z_{t+1}) + r(Z_{t+1})K_{t+1}(Z_t) + \pi(Z_{t+1}) \tag{3.11}$$

where

$$\pi(Z_{t+1}) = \int_0^1 \pi_{t+1}(i)di, \tag{3.12}$$

$K_{t+1}(Z_t)$ is given by (3.2), and the expectations in (3.8) are taken with respect to the equilibrium joint distribution of all future variables.

Consumers' optimal demands for the various consumption and capital goods are determined by the Euler conditions:

$$u'(C_t)\frac{\partial C_t}{\partial c_t(i)}\frac{1}{p_t(i)} = \beta E\left\{\frac{u'(C_{t+1})}{p_{t+1}(j)}\frac{\partial C_{t+1}}{\partial c_{t+1}(j)} \cdot (r_{t+1} + 1 - \delta)\right\}, \quad i, j \in [0, 1] \tag{3.13}$$

where the expectations are taken with respect to the realization of Z_{t+1}, conditioned on Z_t. Equation (3.13) equates the marginal utility derived from an additional unit of income spent on consumption good i at time t, with the discounted marginal utility of spending the (total) returns from saving that income on good j at the next period. Notice that the Euler condition must hold for all pairs of goods $(i, j) \in [0, 1]$.

3.2. Producers

As denoted above, let $[0, \lambda_t]$ be the index set of producers in the $R\&D$-performing sector, and let $(\lambda_t, 1]$ be the non-$R\&D$ sector, with $0 \leq \lambda_t \leq 1$.

Producers are aware of the evolution of the default technology in the economy over time, in (2.9). In particular, each $R\&D$ performing producer realizes that it has a negligible impact on the default technology of the subsequent period. Consequently, producers solve a *non-dynamic* problem.

Production of goods is characterized by constant returns to scale for each individual producer, (regardless of whether $R\&D$ is performed by that producer or not), and therefore unit production cost is independent of the quantity produced. Competition among capital goods producers results in equality between the common price of capital goods, normalized to unity, and the marginal production cost. Consequently,

$$MC_t(i) = 1,\ i \in [0, \lambda_t] \tag{3.14}$$

Different consumption goods are not perfect substitutes. Therefore, producers enjoy some degree of monopoly power in the consumption good market. Thus, every producer needs to set the price for its consumption good version, and decide whether to engage in $R\&D$ or not. While not all producers make the same choice, there is enough symmetry in the model to assume that all those that opt for $R\&D$ will pursue this activity at the same level. Accordingly, we consider equilibria which are symmetric within each of the two classes of producers.

Consumption Goods Prices

Each producer has some monopoly power in the market for its own consumption good, and none in the saving goods market. Denote the marginal revenue function for good i, given its price, by $MR(i, p)$. Accordingly, consumption goods prices must satisfy:

$$MR[i, p_t(i)] = MC_t(i),\ \forall i \in [0, 1] \tag{3.15}$$

Note that since the marginal cost is unitary for $i \in [0, \lambda_t]$, (3.15) pins down the consumption good price only for $R\&D$ performing producers.

R&D-performing Producers

Treating factor prices as given, each *R&D*-performing producer allocates capital and labor between production and *R&D* so as to equate their marginal products in these two activities. Denoting the technology generated by producer i by $\psi_t(i)$, we have for $i \in [0, \lambda_t]$:

$$\begin{aligned} & F_K\left[\psi_t(i), k_t^y(i), l_t^y(i)\right] \\ = \ & F_A\left[\psi_t(i), k_t^R(i), l_t^R(i)\right] \cdot G_K\left[k_t^R(i), l_t^R(i), B_t\right] \end{aligned} \tag{3.16}$$

and:

$$\begin{aligned} & F_L\left[\psi_t(i), k_t^y(i), l_t^y(i)\right] \\ = \ & F_A\left[\psi_t(i), k_t^y(i), l_t^y(i)\right] \cdot G_L\left[k_t^R(i), l_t^R(i), B_t\right] \end{aligned} \tag{3.17}$$

Equilibrium Factor Prices

Factor mobility across industries implies that equilibrium rental and wage rates at time t should equal the value of marginal products of capital and labor in both the *R&D* and the non-*R&D* sectors. Thus, for all $i \in [0, 1]$:

$$r_t = MR\left[i, p_t(i)\right] \cdot F_K\left[A_t(i), k_t^y(i), l_t^y(i)\right] \tag{3.18}$$

$$w_t = MR\left[i, p_t(i)\right] \cdot F_L\left[A_t(i), k_t^y(i), l_t^y(i)\right] \tag{3.19}$$

Equilibrium Profits

Capital good producers sell these goods at cost. Consequently, profits are generated only by sale of consumption goods. Producers engaged in *R&D* have to pay for the resources employed in *R&D*, and benefit from a lower unit production cost. Therefore, producer's profits are given by:

$$\pi_t(i) = \begin{cases} \left[p_t(i) - MC_t(i)\right] c_t(i) - r_t k_t^R(i) - w_t l_t^R(i), \ i \in [0, \lambda_t] \\ \left[p_t(i) - MC_t(i)\right] c_t(i), \ i \in (\lambda_t, 1] \end{cases} \tag{3.20}$$

In an equilibrium with endogenous $R\&D$ participation decisions:

$$\pi_t(i) = \pi_t(j),\ i \in [0, \lambda_t],\ j \in (\lambda_t, 1]. \tag{3.21}$$

3.3. Equilibrium

Given initial technology and capital stock, (A_0, K_1), an equilibrium consists of contingent sequences of production of consumption and capital goods, factor allocations to production and $R\&D$, and pricing decisions by each producer, $\{c_t(i),$ $k_{t+1}(i),\ k_t^y(i),\ l_t^y(i),\ k_t^R(i),\ l_t^R(i),\ p_t(i),\ i \in [0,1]\}$, and consumption and saving decisions by consumers, $(C_t, K_{t+1} - (1-\delta) K_t)$, which solve the producer and consumer optimization problems and satisfy the overall resource constraints. In addition, with endogenous $R\&D$ participation decisions, producers' segmentation into the $R\&D$ and non-$R\&D$ classes, (λ_t), requires that per-firm profits be equal. Finally, given $R\&D$ decisions at time t, A_t is given by (2.9).

4. A Fully Specified Economy

4.1. The Specification

In this section we adopt particular functional forms which allow us to characterize and compute a stationary equilibrium. We drop the time subscript unless needed.

The composite consumption good is generated by:

$$C = \Phi\left(c(\cdot)\right) = \int_{i=0}^{1} c(i)^{\nu} di,\ 0 < \nu < 1. \tag{4.1}$$

The utility function is specified as:

$$u(C) = \frac{1}{\nu} \ln C. \tag{4.2}$$

The production function of output is:

$$F(A,k,l) = TA^{\alpha}k^{\gamma}l^{1-\gamma},\ 0<\gamma<1,\ 0<\alpha, \tag{4.3}$$

and the *R&D* technology:

$$G(k^R, l^R, B) = \psi = B\cdot\left(k^R\right)^{\epsilon}\left(l^R\right)^{1-\epsilon},\ 0<\epsilon<1. \tag{4.4}$$

The default technology evolves through time according to:

$$A_t = \lambda_t^{\mu}\int_0^{\lambda_t}\frac{\psi_t(i)}{\lambda_t}di + (1-\lambda_t^{\mu})A_{t-1}. \tag{4.5}$$

where $\psi_t(\cdot)$ is the new technology developed at time t by producer i, $i\in[0,\lambda_t]$, and μ is an inverse measure of the rate at which technological innovations diffuse from the *R&D* performing sector to the entire economy, (higher μ represent a smaller diffusion rate).[13]

Given consumption expenditure, q_c, and consumption good prices, $p(i)$, $i\in[0,1]$, current utility from composite consumption according to (4.1) and (4.2) is maximized by the following demand functions for the consumption goods:

$$c(i) = q_c\cdot\frac{p(i)^{\frac{1}{\nu-1}}}{\int_{j=0}^{1}p(j)^{\frac{\nu}{\nu-1}}dj} \tag{4.6}$$

while the allocation of consumers income to savings and consumption is determined by the Euler condition, (3.13). These demand functions have constant price elasticities, $\frac{1}{\nu-1}$, and imply the marginal revenue function:

$$MR\left[i,p(i)\right] = p(i)\left[\frac{1}{1/(\nu-1)}+1\right] = p(i)\cdot\nu. \tag{4.7}$$

[13]The parameter μ is used below for the purpose of generating a reasonable growth rate in our numerical examples. It may be used to study the impact of higher diffusion rates on the extent of *R&D* and related issues. See the concluding section for further details.

Finally, we impose a restriction on the parameters in (4.3) and (4.4):

$$\alpha\epsilon + \gamma = 1, \tag{4.8}$$

which can be called "constant *total* returns to capital". This restriction enables us to concentrate on a particular class of equilibria as discussed below.

4.2. Equilibrium

Under the specific functional forms and the additional parametric restriction, the equilibrium can be characterized by a simple state variable defined by:

$$Z_t = (x_t, B_t), \tag{4.9}$$

where x_t is a technology-scaled measure of capital stock given by:

$$x_t \equiv \frac{K_t^{\epsilon}}{A_{t-1}}. \tag{4.10}$$

The following features greatly simplify the complete characterization of the equilibrium :

(a) All producers engaged in $R\&D$ are fully symmetric, as are all those that opt for the default technology;

(b) All producers engaged in $R\&D$ turn out to allocate fixed shares of the resources at their disposal to $R\&D$:

$$\rho = \frac{k_t^R(i)}{k_t(i)} = \frac{\alpha\epsilon}{\gamma + \alpha\epsilon}, \tag{4.11}$$

$$\eta = \frac{l_t^R(i)}{l_t(i)} = \frac{\alpha(1-\epsilon)}{(1-\gamma) + \alpha(1-\epsilon)}. \tag{4.12}$$

(c) Having superior technologies, only $R\&D$-performing producers supply capital goods.

(d) Given the unitary marginal cost, (3.14), the equilibrium price of a consumption good of an *R&D*-performing producer is $1/\nu$ by (4.7).

The remaining six variables to be determined in equilibrium are:

λ_t = the measure of producers who are engaged in *R&D*;

λ_{Kt} = the share of the total capital stock in the economy employed by all *R&D*-performing producers;

λ_{Lt} = the share of labor employed by all *R&D*-performing producers;

p_{2t} = the relative price of a typical consumption good from the non-*R&D* class of producers in terms of the capital good;

θ_t = the ratio of consumption good produced by a typical *R&D*-performing producer to the consumption good produced by a non-*R&D* producer;

σ_t = the share of the output of an *R&D*-performing producer designated to become capital, defined as:

$$\sigma_t \equiv \frac{k_{t+1}(i)}{y_t(i)} = 1 - \frac{c_t(i)}{y_t(i)}, \; i \in [0, \lambda_t]. \tag{4.13}$$

The equilibrium conditions determining the variables $\lambda_t, \lambda_{Kt}, \lambda_{Lt}, p_{2t}, \theta_t, \sigma_t$ and x_{t+1}, given x_t and the realization of B_t, are derived in Appendix A.

In equilibrium, the fraction of output designated as capital by every producer matches the consumers' behavior. Therefore, we may also refer to σ_t as the *physical saving rate.* This rate is rather different from the *measured saving rate,* s_t, as implied by NIPA. The latter is defined by the ratio of the *value* of saving, to the *value* of total income using market prices, (q_t in (3.10)), which under our specification can be written as:

$$s_t = \frac{\int_0^{\lambda_t} k_{t+1}(i)di}{q_t} = \frac{\lambda_t \sigma_t y_{1t}}{\lambda_t \left[\frac{1}{v}(1-\sigma_t)y_{1t} + \sigma_t y_{1t}\right] + (1-\lambda_t)p_{2t}y_{2t}} \tag{4.14}$$

where y_{1t} and y_{2t} are total output levels of a typical $R\&D$-performing producer and a typical non-$R\&D$ producer, respectively.

4.3. Dynamic Properties

There are altogether seven equilibrium variables to be determined at each period. We let z_t denote the 5-elements vector,

$$z_t = (\lambda_t, \lambda_{Kt}, \lambda_{Lt}, p_{2t}, \theta_t). \tag{4.15}$$

The components of z_t are determined each period by profit maximizing behavior of producers, given the state (x_t, B_t) and parametrized by the endogenous variable σ_t. The solution is denoted by

$$z_t = H(\sigma_t, x_t, B_t). \tag{4.16}$$

These are the non-dynamic components of the equilibrium.

The Euler condition determines the consumers' physical saving, σ_t. That condition involves z_t, z_{t+1}, and σ_{t+1} so that consumers must form expectations on the future values of z and σ. These expectations are computed using the stochastic process governing B in (2.7) and (4.16). We summarize this dynamic relationship by:

$$\sigma_t = Q\left(z_t, E_t\left\{M(\tilde{z}_{t+1}, \tilde{\sigma}_{t+1})\right\}\right), \tag{4.17}$$

where the functions $Q(\cdot)$ and $M(\cdot)$ are implied by the structure of the model.

Finally, the law of motion of x_t is derived from the evolution of the default technology, and is represented as:

$$x_{t+1} = \Lambda(\sigma_t, z_t, x_t, B_t). \tag{4.18}$$

The system consisting of (4.17), (4.18) and (2.7), using repeatedly equation (4.16), constitutes a stochastic dynamic system in (x_{t+1}, σ_t, B_t), $t = 1, 2, ...$, given initial values of $x_1$and B_1. There is no initial condition for σ.

Suppose there exists an equilibrium path in which the state (x_t, B_t) uniquely generates σ_t and z_t for all t. Then, around its non-stochastic steady-state the system must possess the saddle-path property. Failing this (local) condition implies that an equilibrium of the desired form does not exist.

As it turns out, the system is very sensitive to relatively minor perturbations of the underlying parameters. In some cases both eigenvalues of the linearized system are inside the unit circle while in others both are outside the unit circle. We also obtained some systems in which one root is inside and one outside the unit circle, as desired. However, in a modified version of the equilibrium, where some of the endogenous responses are neutralized, we found that the saddle-path property holds for all parameter combinations we tried. In particular, we consider below a restricted equilibrium, in which producers may not shift between the *R&D* and the non-*R&D* sectors in response to changes in the environment. This system does not include the equal profits condition, (3.21), and λ is fixed at its original steady state level. We refer to the unrestricted steady state as the *fully adjusted* equilibrium, and to the restricted one as *partially adjusted.*[14]

[14]We believe that the problem with the fully adjusted equilibrium is the absence of an explicit dynamic process that describes the movement of producers between the *R&D* generating sector and the non-*R&D* generating sector. A possible way to restore the saddle path property is to associate an expicit adjustment costs with the move from one class of producers to the other. This implies a significant complication of the individual producer's decision rules. We do not pusrue this possibility in the present paper.

5. Comparative Steady-States

The system described above is quite complicated, and very little may be said about its general properties. Therefore we use numerical methods to evaluate the system's behavior, focusing first on steady-state effects and turning later to dynamic effects.

We choose to concentrate here on the saving and growth relationships which emerge from our setup.[15] Specifically, we compare the solutions obtained under two changes which are likely to affect saving and growth; First, we change the subjective discount factor, β. This is virtually analogous to directly changing the saving rate in models in which the saving rate is exogenous. Next we change the $R\&D$ productivity parameter, B. Now the impact on saving is indirect, and incorporates complex dynamic considerations. In both cases we find that in steady-states the (measured) saving rate is increasing along with the growth rate. Such long-run co-movements of growth and measured saving rates seem to characterize the data (e.g., Mankiw, Romer and Weil (1992)).

Due to the aforementioned problem of dynamic stability, we report below the results both for a fully adjusted system, in which the size of the knowledge-based industry is allowed to change, and for the partially adjusted system, in which the value of λ is fixed. As it turns out, the saving-growth relationships are qualitatively similar in both cases. Still, the differences between the behavior of the fully adjusted system and that of the partially adjusted one are quite revealing, and are discussed below.

[15] These relationships have been discussed both theoretically and empirically in the context of the debate on the endogenous growth literature, and are therefore relatively well researched (see, for example, Mankiw, Romer and Weil (1992), Jones (1995b), and McGrattan (1998)).

5.1. Parameter Choice

The base-line steady-state we compute applies to the following parameter values: $T = 0.12$, $B = 10$, $\alpha = 0.21$, $\gamma = 0.8$, $\mu = 9, \delta = 0.08$, $\beta = 0.96$, $\nu = 0.2$.

The parameters were chosen under the interpretation of a period as a year. Accordingly, the values of β and δ are the standard ones for yearly data. The remaining parameter values are chosen subject to restrictions the model imposes so as to generate yearly growth and interest rates which are close to those observed in industrial countries (2-3% and 5-6%, respectively).[16]

Specifically, the choice of γ is restricted by bounds the model places on α: from the restriction $\alpha\varepsilon + \gamma = 1$ and $\varepsilon < 1$ we get that $\alpha > 1 - \gamma$. An upper bound on the value of α is obtained from the requirement that there be an interior solution (i.e. - that not all producers choose to engage in $R\&D$). In order to keep the lower bound on α below the upper bound, the value of γ cannot be too low. In fact, the "standard" value of γ, usually around 0.3, is far too low. However, in our setup the value of γ is no longer directly associated with the capital-share in the national income. As can be seen below, the relatively high value our model requires yields a capital share of about 0.4.[17]

The above choices of γ and of α imply a value of 0.952 for ε. This value too may look high. However, it has an implication which is roughly corroborated by

[16] These restrictions still leave some degrees of freedom on the choice of the parameters. However, the results are nunmerically very close once β and δ are set at their yearly values and the interest and growth rates are targeted to about 2% and 6% respectively.

[17] This value may still be a little high. However, our model does not contain a government sector. We believe that having a sizable sector in the economy which is labor intensive would help reduce the capital share in the national income.

the data: the model implies that the share of the labor force allocated to $R\&D$ is $\lambda_L \eta$, which is the share of the labor force employed by the knowledge-based industries times the share of their labor force these industries allocate to $R\&D$. The model implies further that $\eta = 1 - \varepsilon$, which in our case is roughly 0.05. As can be seen in the tables below, we obtain that λ_L is roughly 0.8. Therefore, the share of the labor force engaged in $R\&D$ is predicted to be about 0.04. This number is not far from the share of scientists, engineers and technicians in total employment in the US.[18]

5.2. Changes in β

We report below how the system behaves when β is decreased to 0.95.

[18] In 1996 the Bureau of Labor Statistics estimated that there were 4,885.5 thousand civilian scientitsts, engineers and technicians (Table 1001 of the Statistical Abstracts of the United States, 1998), while civilian employment that year was 126,708 thousand (Table B-35, Economic Report of the President, 1998). The ratio is 0.0386. The number of those *defined* as $R\&D$ scientists and engineers was 789.5 thousands in 1995 (Table 1000 of the Statistical Abstracts of the United States, 1998), amounting to a fraction of merely 0.006 of civilian employment.

Table 1: **Equilibrium Responses to Changes in β**

Variable	$\beta = 0.96$	$\beta = 0.95$	
		fully adjusted	partially adjusted
Growth[a]	1.024	1.004	1.011
Interest[b]	1.066	1.058	1.064
Capital Income Share	0.418	0.355	0.362
Knowledge-based Output Share[c]	0.530	0.499	0.478
s	0.297	0.218	0.228
σ	0.864	0.796	0.821
θ	11.418	4.426	9.320
p_2	35.079	16.437	29.820
λ	0.234	0.293	0.234
λ_K	0.820	0.774	0.769
λ_L	0.793	0.742	0.737
Non-*R&D* Profits[d]	0.334	0.606	0.459
R&D Profits[e]	0.334	0.606	0.509
x	1636	59	806

[a]One plus the growth rate.
[b]Gross rate of return on capital.
[c]Value-based.
[d]Per firm in the non-*R&D* performing sector.
[e]Per firm in the *R&D* performing sector.

Table 1 reveals that the qualitative reaction of the system to change in β do not depend on whether it is allowed to fully adjust or not; The increased subjective discount factor decreases the growth rate and both the physical and the measured saving rates. The changes in the endogenous variables tend to be larger (in absolute value) when the system is allowed to fully adjust, except for the changes in the allocation of resources in the economy (λ_K and λ_L).

The reduction of β reduces the share of the knowledge-based industries in the employment of both factors of production. However, the size of the knowledge-based sector increases when λ is allowed to change. This fact is reflected by the higher profit that sector enjoys in the partially adjusted system.

It is noteworthy that the decline in the growth rate is bigger when the system fully adjusts. This is due to the fact that all the variables that affect the growth rate (see Appendix A) move in a consistent way. The physical saving rate, σ, and the shares of the knowledge-based industries in the employment of resources, λ_K and λ_L, are positively related to the growth. The value of these variables falls. On the other hand, the size of the knowledge-based sector, λ, is *negatively* related to the growth rate. This reflects the fact that as λ increases, ceteris paribus, the resources allocated to $R\&D$ are spread over a larger "number" of $R\&D$ generating producers, which decreases the knowledge generated by every individual producer. Since the knowledge impact on the economy as a whole depends on an *average* of all $R\&D$ "outputs" and not on their sum, the impact of increasing λ on growth is negative.

The impact of decreasing β on the value of x (which is K^{ϵ}/A) is negative and very large. This means that along the transition path K^{ϵ} must fall far more than A. Intuitively, this phenomenon is related to the sluggish adjustment of the level of "knowledge" (A) when λ is rather small and the diffusion parameter, μ, is rather big, (see (4.5))

The relative price of the consumption good produced by the non-$R\&D$ sector, p, declines with β. Therefore, the growth rate and that relative price move together. This is consistent with the observed negative relationship between growth rates and the relative price of equipment reported in the literature (see DeLong and Summers (1991) and Jones (1994)).

Finally, the value-based share of the knowledge-based industries in total output is about 0.5. This value is very close to the actual figure of 0.46 reported in the introduction.

5.3. Changes in B

Here, B is changing from its base-value of 10 to 5.[19] All other parameter values are being held fixed.

Table 2: **Equilibrium Responses to Changes in** B

Variable	$B = 10$	$B = 5$	
		fully adjusted	partially adjusted
Growth[a]	1.024	1.005	1.003
Interest[b]	1.066	1.047	1.045
Capital Income Share	0.418	0.387	0.384
Knowledge-based Output Share[c]	0.530	0.484	0.487
s	0.297	0.259	0.256
σ	0.864	0.852	0.847
θ	11.418	9.029	6.912
p_2	35.079	29.072	23.478
λ	0.234	0.219	0.234
λ_K	0.820	0.787	0.786
λ_L	0.793	0.759	0.756
Non-*R&D* Profits[d]	0.334	0.380	0.415
R&D Profits[e]	0.334	0.380	0.401
x	1636	1303	504

[a]One plus the growth rate.
[b]Gross rate of return on capital.
[c]Value-based.
[d]Per firm in the non-*R&D* performing sector.
[e]Per firm in the *R&D* performing sector.

Table 2 reveals that when the *R&D* productivity is permanently decreased, growth, as well as the physical and measured saving rates decrease. The impact

[19]The relatively large change in B is needed to make the impact numerically noticeable.

on the endogenous variables is larger (in absolute value) when the system is not allowed to fully adjust. Now, the profits in the knowledge-based sector fall as a result of the reduced $R\&D$ productivity, and producers want to shift out of that sector. When they are allowed to do so, there is a (small) improvement in the growth rate, because a smaller number of $R\&D$ generating producers share the resources allocated to the knowledge-based sector. The decline in the value of x is larger in the partially adjusted system because of the greater decline in σ, which implies a faster decline in physical accumulation along the transition path. The relative price of the non-$R\&D$ consumption good moves together with the growth rate in this experiment as well.

6. Dynamics: Technology Shocks, Growth, Saving and Prices

The comparative steady-states are valid to conduct "long-run" comparisons of two different economies characterized by different parameters. In the "short-run" the saving (or investment)-growth relationships in the data may be quite different. In fact, these short-run correlations are not necessarily positive, an observation that has been used to criticize the endogenous-growth models (Jones (1995b)). We show here that our setup may be consistent with such observations. Specifically, in our model the short-term correlations between growth and saving may be either positive or negative, depending on the size of the serial correlation in the process generating the productivity $R\&D$.[20] We also show that the correlation between

[20] McGrattan (1998) also shows that the short-term negative association of growth and saving may be consistent with an endogenous growth model. Her explanation depends on particular changes in taxes on different types of capital. While this expalanation may be consistent with US data, it is doubtful whether it applies to other countries.

growth and the relative price of the non-$R\&D$ consumption good depends on that serial correlation.

To provide some background to the results we obtain from our model, we first report some correlations found in actual data. Table 3 reports sample correlations in the yearly data for the G-7 countries between 1950 and 1992.[21] We compute the correlations of the deviation from the average investment share of GDP, i, as well as the saving share of GDP, s, with the deviations from the average growth rate of per-capita GDP, g.[22] The table contains the contemporaneous correlations, as well as the correlations of current saving and investment measures with the following year's growth, and current growth with saving or investment of the following year. For reference we also report the average share of the current account surplus of GDP (in percentage points) and its standard deviation.[23]

[21] The data are taken from the Penn-World Tables, Mark 5.6.

[22] The correlations of the actual data are very close to those of the deviations from the averages.

[23] The growth rate for year t is computed as $(y_t + y_{t+1})/(y_{t-1} + y_t) - 1$, where y_t is year t per-capita real GDP. The investment share of GDP, i, is reported in the Penn-World Tables. The saving share, s, is one minus the sum of the reported consumption share of GDP and the government share of GDP. The share of the current account surplus of GDP is one minus the sum of the consumption share, invetment share and government share.

Table 3: **Data Correlations between Saving Measures and Growth**

	Saving measure	$cor(\bullet_t, g_t)$	$cor(\bullet_t, g_{t+1})$	$cor(\bullet_{t+1}, g_t)$	Current account (Std. dev.)
Canada	i	-0.43	-0.77	0.03	1.2
	s	0.06	-0.32	0.43	(1.4)
US	i	0.37	-0.25	0.79	-0.7
	s	0.11	-0.23	0.42	(1.0)
Japan	i	-0.36	-0.50	0.14	-1.9
	s	-0.48	-0.58	-0.30	(2.3)
France	i	0.07	-0.14	0.31	-1.5
	s	-0.08	-0.21	0.14	(1.1)
Germany	i	0.32	0.04	0.53	0.6
	s	0.44	0.21	0.62	(1.3)
Italy	i	0.50	0.27	0.69	-1.6
	s	0.50	0.28	0.65	(1.3)
UK	i	0.01	-0.23	0.37	0.0
	s	0.04	-0.07	0.25	(1.2)

In most cases, the contemporaneous correlation in the data between either the investment share or the saving share and growth is positive, even if small. Japan is a striking exception. The sign pattern of the correlation between the current investment or saving shares and next year's growth is mixed, where in five of the seven countries these signs are negative: an increase in current saving or investment is associated with a decline in future growth. In contrast, all (but one) of the correlations between current growth and next year's saving or investment shares are positive.

Next we report the corresponding correlations generated by our model. To compute these correlations, we linearize the dynamic system around the deterministic steady-state obtained with $T = 0.12$, $B = 10$, $\alpha = 0.21$, $\gamma = 0.8$, $\mu = 9, \delta = 0.08$, $\beta = 0.96$, $v = 0.2$, used for the computations in Tables 1 and 2 above.

We denote by $\hat{X}_t$ the (level) deviation of any variable X_t from its non-stochastic steady state value, and study the dynamic and stochastic behavior of these deviations. In particular, we feed through the linearized system realizations of a random process which is generating deviations of the $R\&D$ productivity parameter, $\hat{B}$, from their mean value of 10, specified as:

$$\hat{B}_t = r \cdot \hat{B}_{t-1} + u_t, \;\; -1 < r < 1 \tag{6.1}$$

where u_t is drawn from a normal distribution with a mean of zero and a standard deviation of 0.01. Clearly,

$$E_t[\hat{B}_{t+1}] = r \cdot \hat{B}_t.$$ [24]

This expected value is used in the (linearized) Euler equation, as well as the relationship between the expected deviations of the variables contained in z (see (4.15) above) which are induced by it.

Since the model does not possess the saddle path property when the size of the $R\&D$ generating sector is allowed to change, we fix λ at its steady-state value, omit the profit equalization condition from the set of equilibrium equations, and compute the saddle-path.

We run the model 100 times, each run with 100 periods. Table 4 below reports the average (and standard deviations) of the correlations between the deviations of the measured saving rate, $\hat{s}_t$, with the deviations of the growth rate, $\hat{g}_t$. We also report those correlations with the growth rate of the following period, as well as the correlations of $\hat{\sigma}_{t+1}$ and $\hat{s}_{t+1}$ with $\hat{g}_t$ for different values of the serial correlation r. In addition, the table reports correlations between the price of the consumption good produced by the non-$R\&D$ sector and the growth rate.

Table 4: **Simulated Corrleations**
(std. deviations in brackets)

r	$cor(\hat{s}_t, \hat{g}_t)$	$cor(\hat{s}_t, \hat{g}_{t+1})$	$cor(\hat{s}_{t+1}, \hat{g}_t)$	$cor(\hat{p}_t, \hat{g}_t)$
0.	-0.667 (0.042)	0.001 (0.096)	0.757 (0.029)	0.286 (0.143)
0.5	-0.398 (0.058)	-0.199 (0.089)	0.618 (0.028)	-0.383 (0.062)
0.9	0.298 (0.138)	0.245 (0.146)	0.741 (0.028)	-0.661 (0.038)

The contemporaneous correlation between measured saving and growth depends on the serial correlation in B. When that serial correlation is sufficiently low, saving and growth are *negatively* correlated. Only for high values of the serial correlation, measured saving and growth are contemporaneously positively correlated.

This phenomenon is not easily explained, since the measured saving rate is not a behavioral parameter in this model. However, some insight may be gained by considering an important component of the measured saving rate, namely the physical saving rate. A positive serial correlation in B induces two, conflicting, forces. First, there is an income effect. A positive shock to current B implies higher future income, which in turn increases current consumption and reduces current physical saving. On the other hand, there is a substitution effect. Any increase in the current stock of physical capital will yield higher output in the future, as the higher future *R&D* productivity will be applied to that increased physical capital stock. This force tends to increase current physical saving. As it turns out, the income effect dominates as long as the serial correlation in B is not too high. Once this serial correlation becomes high, the substitution effect dominates. This, in turn, causes the change in the sign of the correlation between the measured saving rate and growth.

In Table 3 we see that the sign pattern of the contemporaneous correlation between the saving (or investment) rate and growth among the G-7 countries is quite mixed. Although some of the data points are characterized by quite low correlations in absolute value while the model's average correlations tend to be large, the observed sign pattern is not at odds with the predictions of the model.

The current measured saving rate and next period's growth rate are unrelated when the serial correlation in B is zero, are weakly negatively correlated when the serial correlation in B is not too high, and are positively correlated when that serial correlation is high. Again, the analogous empirical sign pattern is mixed. However, the model does not quite match the patterns of the correlation found in the data. In particular, in the data the correlation between current saving or investment rates and future growth is smaller than that between current saving and current growth. The model does not deliver this feature.

The model matches well the correlation between current growth and future saving (or investment) rates. In the data and in the model, for all values of the serial correlation in B, current growth and future saving (or investment) rates tend to be highly positively correlated.

Finally, the correlation between growth and the price of the non-$R\&D$ consumption good is positive for the lower serial correlations in B and negative when that serial correlation is high. The empirical findings that capital goods prices are negatively correlated with growth rates are therefore consistent with our model in the former case.[28]

[28] See DeLong and Summers (1991) and Jones (1994). Remember that the capital good is the numeraire, and the relative price of the consumption good produced by "knowledge-industries"

7. Conclusion

Most of the non-defense $R\&D$ is carried out by firms which are directly linked to the markets for final goods. This association is mostly noticed in the production of investment goods, but it exists also in the market for consumption goods. The model we have constructed mimics this reality. In particular, by introducing a quasi-fixed cost into $R\&D$ activities, it generates two classes of producers: one in which producers choose to allocate resources to the creation of new technologies, and one in which producers choose to use the commonly-known "state-of-the-art" technology. The cost advantage of the memebers of the first group implies that the capital good is produced solely by them. Naturally, the price of the consumption goods produced by the $R\&D$ sector is lower than those produced by the members of the non-$R\&D$ sector.

The model mimics reality also on the quantitative dimension. Specifically, parameter values which generate reasonable growth rates and interest rates, yield also quite reasonable shares of capital in national income and of $R\&D$-based output in GDP. Furthermore, the model can accomodate the commonly-observed positive long-run correlations between saving rates and growth, as well as short-run negative correlations between these variables which characterize some countries. Likewise, the model is consistent with the negstive relationship between the relative price of capital goods and growth observed in the data.

There are numerous extensions of the model we have not pursued. One such extension concerns the recent contributions to the literature on economic growth that have emphasized the speed at which new technologies are adopted throughout

is fixed at $1/\nu$.

an economy as a key feature explaining that economy's performance (Prescott (1998)). Our model can potentially shed some light on this important issue, exploring further the elements that affect the rate at which the non-$R\&D$ sector adopts the technologies developed in the $R\&D$ sector.

The most important extensions relate, of course, to policy. The model can be used to explore the standard questions on subsidies to $R\&D$. Similarly, one may analyze the impact of subsidies to saving. All such measures unleash complex responses, related to the number of producers engaged in $R\&D$ and to the amount of resources they employ, that need to be explored.

The interesting new angle about policy making highlighted by our model is the relationship between the composition of final uses and economic growth. A government may affect the development of its economy by creating demand for "$R\&D$-intensive" goods, (see Nelson (1968)) for an early discussion). This question needs to be explored both empirically and theoretically.

A. Equilibrium Conditions for Particular Functional Forms

Here we derive the equilibrium conditions for the particular functional forms used in our simulations, in terms of the variables $\lambda_t, \lambda_{Kt}, \lambda_{Lt}, p_{2t}, \theta_t$, σ_t, and x_t, which are explained in the text.

With complete symmetry within each of the two sectors, and by (3.14) and (3.15), each $R\&D$-performing producer sets the price of his consumption good to $\frac{1}{\nu}$, while non-$R\&D$ producers set a higher price, p_{2t}, reflecting their higher marginal costs:

$$p_t(i) = \begin{cases} 1/\nu, & i \in [0, \lambda_t] \\ p_{2t}, & i \in [\lambda_t, 1] \end{cases} . \tag{A.1}$$

Likewise, for any other firm-specific variable we let:

$$m_t(i) = \begin{cases} m_{1t}, & i \in [0, \lambda_t] \\ m_{2t}, & i \in (\lambda_t, 1] \end{cases} ,$$

where $m_t(i)$ can be consumption, $(c_t(i))$, total output, $(y_t(i))$, capital, $(k_t(i) = k_t^y(i) + k_t^R(i))$, and labor, $(l_t(i) = l_t^y(i) + l_t^R(i))$, for producer i.

Denote relative consumption across sectors by:

$$\frac{c_{1t}}{c_{2t}} = \theta_t. \tag{A.2}$$

Using (4.6) and the fact that $p_{1t}(i) = \frac{1}{\nu}$ for $i \in [0, \lambda_t]$, we have:

$$\theta_t = \left(\frac{1/\nu}{p_{2t}} \right)^{1/(1-\nu)} . \tag{A.3}$$

Equating the marginal products of capital and labor in both goods production and $R\&D$, we get from (3.16) and (3.17) the constant shares of factors allocated to R&D, reported in the text as (4.11) and (4.12).

We denote by λ_{Kt} and λ_{Lt} the share of capital and labor, respectively, employed (in total) by the $R\&D$-performing sector, so that:

$$\lambda_{Kt} \equiv \frac{\int_{i=0}^{\lambda_t} k_t(i) di}{K_t} \tag{A.4}$$

$$\lambda_{Lt} \equiv \int_{i=0}^{\lambda_t} l_t(i) di$$

Equating the marginal value product of both factors across the two sectors, we get from (3.18) and (3.19) that capital labor ratios in production of goods are the same for both sectors:

$$\frac{k_{1t} - k_{1t}^R}{l_{1t} - l_{1t}^R} = \frac{k_{2t}}{l_{2t}} \tag{A.5}$$

or, using definition (A.4) and the fixed allocation rules (4.11) and (4.12):

$$\frac{(1-\rho)\lambda_{Kt}}{(1-\eta)\lambda_{Lt}} = \frac{1-\lambda_{Kt}}{1-\lambda_{Lt}} \tag{A.6}$$

The marginal revenues from (4.7) are given by $MR(i) = 1$ for producers in the $R\&D$ sector, $(i \in [0, \lambda_t])$, and by $MR(i) = vp_{2t}$ for non-$R\&D$ producers, $(i \in (\lambda_t, 1])$, . Substituting these values in the marginal revenue products, (3.18) and (3.19), and using the equal capital-labor ratios in production across sectors, we get:

$$\nu p_{2t} = \left\{ \frac{B_t \left(\rho \frac{\lambda_{Kt}}{\lambda_t} K_t\right)^{\epsilon} \left(\eta \frac{\lambda_{Lt}}{\lambda_t}\right)^{1-\epsilon}}{A_{t-1}} \right\}^{\alpha}. \tag{A.7}$$

Since only $R\&D$-performing firms produce capital goods, we let σ_t denote the fraction of the output of such a firm designated for that purpose, so that:

$$c_{1t} = (1-\sigma_t)\, y_{1t}, \tag{A.8}$$

and the total capital stock in the economy evolves according to:

$$K_{t+1} = \lambda_t \sigma_t y_{1t} + (1-\delta)\, K_t.$$

The (gross) rate of return on capital held from t to $t+1$, R_{t+1},is:

$$\begin{aligned} R_{t+1} &= r_{t+1} + (1-\delta) = \frac{\gamma y_{1t+1}}{k_{1t+1}^{y}} + (1-\delta) = \frac{\gamma y_{1t+1}}{(1-\rho)\frac{\lambda_{Kt+1}K_{t+1}}{\lambda_{t+1}}} + (1-\delta) \\ &= \frac{\gamma \lambda_{t+1} y_{1t+1}}{(1-\rho)\lambda_{Kt+1}\left[\sigma_t \lambda_t y_{1t} + (1-\delta)\, K_t\right]} + (1-\delta). \end{aligned} \tag{A.9}$$

The composite consumption good is given by:

$$C_t = \int_{i=0}^{1} c_t(i)^{\nu} di = \lambda_t c_{1t}^{\nu} + (1-\lambda_t)\, c_{2t}^{\nu},$$

with

$$u\left(C_t\right) = \frac{1}{\nu} \ln C_t = \frac{1}{\nu} \ln \left[\lambda_t c_{1t}^{\nu} + (1-\lambda_t)\, c_{2t}^{\nu}\right] \tag{A.10}$$

The Euler equation (3.13), for the $R\&D$ sector goods becomes:

$$\frac{c_{1t}^{\nu-1}}{\nu C_t} = \beta E_t \left\{ \frac{c_{1t+1}^{\nu-1}}{\nu C_{t+1}} \times R_{t+1} \right\}$$

which can be written, using (A.10), (A.9), and (A.2), as:

$$\frac{\lambda_t \theta_t^{\nu}}{(1-\sigma_t)\left[\lambda_t \theta_t^{\nu} + (1-\lambda_t) c_{2t}^{v}\right]} = \beta E_t \left\{ \frac{\lambda_{t+1}\theta_{t+1}^{\nu}}{(1-\sigma_{t+1})\left[\lambda_{t+1}\theta_{t+1}^{\nu} + (1-\lambda_{t+1})\right]} \times \right.$$
$$\left. \left[\frac{\gamma}{(1-\rho)\lambda_{Kt+1}\left[\sigma_t + (1-\delta)/H_{1t}\right]} + \frac{(1-\delta)}{H_{2t+1}} \right] \right\}, \tag{A.11}$$

where

$$H_{1t} \equiv \frac{\lambda_t y_{1t}}{K_t} \tag{A.12}$$
$$H_{2t+1} \equiv \frac{\lambda_{t+1} y_{1t+1}}{\lambda_t y_{1t}}$$

In turn, the last two ratios can be written as:

$$H_{1t} = \frac{\lambda_t}{K_t} T B^{\alpha} \left(\rho \frac{\lambda_{Kt}}{\lambda_t} K_t\right)^{\alpha\varepsilon} \left(\eta \frac{\lambda_{Lt}}{\lambda_t}\right)^{\alpha(1-\varepsilon)} \left[(1-\rho)\frac{\lambda_{Kt}}{\lambda_t} K_t\right]^{\gamma} \left[(1-\eta)\frac{\lambda_{Lt}}{\lambda_t}\right]^{1-\gamma}$$

so that with $\alpha\varepsilon + \gamma = 1$ we have:

$$H_{1t} = T B^{\alpha} \rho^{\alpha\epsilon} \eta^{\alpha(1-\epsilon)} (1-\rho)^{\gamma} (1-\eta)^{1-\gamma} \frac{\lambda_{Kt}(\lambda_{Lt})^{\alpha}}{\lambda_t^{\alpha}}, \tag{A.13}$$

and the first line in (A.12) becomes:

$$\lambda_t y_{1t} = T B^{\alpha} \rho^{\alpha\epsilon} \eta^{\alpha(1-\epsilon)} (1-\rho)^{\gamma} (1-\eta)^{1-\gamma} \cdot K_t \frac{\lambda_{Kt} {\lambda_{Lt}}^{\alpha}}{\lambda_t^{\alpha}}. \tag{A.14}$$

Similarly, using (A.14), we get:

$$H_{2t+1} = T B^{\alpha} \rho^{\alpha\epsilon} \eta^{\alpha(1-\epsilon)} (1-\rho)^{\gamma} (1-\eta)^{1-\gamma} \sigma_t \frac{\lambda_{Kt+1} {\lambda_{Lt+1}}^{\alpha}}{\lambda_{t+1}^{\alpha}} +$$
$$(1-\delta) \frac{\lambda_{Kt+1} {\lambda_{Lt+1}}^{\alpha} \lambda_t^{\alpha}}{\lambda_{t+1}^{\alpha} \lambda_{Kt} {\lambda_{Lt}}^{\alpha}}. \tag{A.15}$$

To insure market clearing we derive the demand for consumption goods from the non-$R\&D$ sector as follows. Consumers' income at t, q_t, is simply total output evaluated at market prices, so that consumption expenditure is given by:

$$
\begin{aligned}
q_{ct} &= q_t - s_{t+1} \\
&= \lambda_t \left[\frac{1}{v} c_{1t} + (y_{1t} - c_{1t}) \right] + (1-\lambda_t) p_{2t} y_{2t} - \sigma_t \lambda_t y_{1t} \\
&= y_{1t} \left\{ \lambda_t \left[\left(\frac{1}{\nu} - 1 \right) (1-\sigma_t) + 1 \right] + (1-\lambda_t) p_{2t} \frac{(1-\sigma_t)}{\theta_t} - \sigma_t \lambda_t \right\}
\end{aligned}
\tag{A.16}
$$

By (4.6), the demand for c_{2t} is:

$$
c_{2t} = q_{ct} \frac{p_{2t}^{\frac{1}{1-\nu}}}{\lambda_t \left(\frac{1}{\nu}\right)^{\frac{\nu}{1-\nu}} + (1-\lambda_t) p_{2t}^{\frac{\nu}{1-\nu}}} \tag{A.17}
$$

From the production side, the supply of consumption goods by the non-$R\&D$ sector is:

$$
c_{2t} = A_{t-1}^{\alpha} k_{2t}^{\gamma} l_{2t}^{1-\gamma}.
$$

To express the output of c_{2t} in terms of y_{1t}, multiply and divide the last expression by both sides of (A.7), and use (A.5) to get:

$$
\begin{aligned}
c_{2t} &= A_{t-1}^{\alpha} k_{2t}^{\gamma} l_{2t}^{1-\gamma} \frac{1}{\nu p_{2t}} \left\{ \frac{\psi_t(i)}{A_{t-1}} \right\}^{\alpha} \\
&= \frac{y_{1t}}{\nu p_{2t}} \left(\frac{(1-\lambda_{Lt}) / (1-\lambda_t)}{\left[(1-\eta) \lambda_{Lt} \right] / \lambda_t} \right)^{1-\gamma}
\end{aligned}
\tag{A.18}
$$

Equating the demand and supply for c_{2t} we get:

$$
\begin{aligned}
& \frac{\lambda_t (1-\lambda_{Lt})}{(1-\lambda_t) \nu p_{2t} (1-\eta) \lambda_{Lt}} \\
= \; & \frac{p_{2t}^{\frac{1}{1-\nu}} \left\{ \lambda_t \left[\left(\frac{1}{\nu} - 1 \right) (1-\sigma_t) + 1 - \sigma_t \right] + (1-\lambda_t) p_{2t} \frac{(1-\sigma_t)}{\theta_t} \right\}}{\lambda_t \left(\frac{1}{\nu}\right)^{\frac{\nu}{1-\nu}} + (1-\lambda_t) p_{2t}^{\frac{\nu}{1-\nu}}}
\end{aligned}
\tag{A.19}
$$

In a fully-adjusted equilibrium, profits per firm must be equal across the two sectors. The capital good in the $R\&D$ sector is sold at cost, while the profit margin for the consumption goods is $\left(\frac{1}{v}-1\right)$. Hence, profits are:

$$\begin{aligned}\pi_{1t} &= (\frac{1}{\nu}-1)c_{1t}-\left(r_t k_{1t}^R+w_t l_{1t}^R\right)\\ &= y_{1t}\left\{(\frac{1}{\nu}-1)(1-\sigma_t)-\frac{\rho}{1-\rho}\gamma-\frac{\eta}{1-\eta}(1-\gamma)\right\}.\end{aligned}$$

Likewise, profits in the non-$R\&D$ sector are, using (A.18),

$$\begin{aligned}\pi_{2t} &= c_{2t}p_{2t}(1-\nu)\\ &= \frac{(1-\nu)}{\nu}\frac{\lambda}{(1-\lambda)}\frac{1-\lambda_{Lt}}{(1-\eta)\lambda_{Lt}}\frac{y_{1t}}{\lambda}.\end{aligned}$$

Equating π_1 with π_2 we get:

$$(\frac{1}{\nu}-1)(1-\sigma_t)-\frac{\rho\gamma}{1-\rho}-\frac{\eta(1-\gamma)}{1-\eta}=(1-\nu)\frac{\lambda_t}{(1-\lambda_t)}\frac{1-\lambda_{Lt}}{(1-\eta)\lambda_{Lt}}. \tag{A.20}$$

At this stage, we have defined a time t equilibrium in terms of five variables: $\lambda_t, \lambda_{Kt}, \lambda_{Lt}, p_{2t}$, and θ_t, given (K_t, A_{t-1}), via five equations: (A.3), (A.6), (A.7), (A.19), and (A.20). The physical saving rate at t depends on time $t+1$ variables, according to the Euler equation, (A.11). Factor allocations to $R\&D$ and production of goods by $R\&D$-performing producers are given by (4.11) and (4.12), and are totally independent of any other variable.

As can be seen from the aforementioned five equations, the only relevant part of the state is $\frac{K_t^\epsilon}{A_{t-1}}$, which enters only via (A.7). To determine the evolution of this state variable, define:

$$x_t = \frac{K_t^\epsilon}{A_{t-1}}$$

Given (B_t, x_t), we can compute x_{t+1} from $(\lambda_t, \lambda_{Kt}, \lambda_{Lt}, \theta_t, p_{2t})$ according to:

$$\begin{aligned}x_{t+1} &= \frac{\left[\sigma_t\lambda_t y_{1t}+(1-\delta)K_t\right]^\epsilon}{A_t} \qquad \text{(A.21)}\\ &= \frac{\left[\sigma_t B_t^\alpha\left(\rho\frac{\lambda_{Kt}}{\lambda_t}\right)^{\alpha\epsilon}\left(\eta\frac{\lambda_{Lt}}{\lambda_t}\right)^{\alpha(1-\epsilon)}\lambda_{Kt}^\gamma\lambda_{Lt}^{1-\gamma}+(1-\delta)\right]^\epsilon\frac{K_t^\epsilon}{A_{t-1}}}{\lambda_t^\mu B_t\left(\rho\frac{\lambda_{Kt}}{\lambda_t}\right)^\epsilon\left(\eta\frac{\lambda_{Lt}}{\lambda_t}\right)^{1-\epsilon}\frac{K_t^\epsilon}{A_{t-1}}+(1-\lambda_t^\mu)}\end{aligned}$$

where the last equality follows from $\alpha\epsilon + \gamma = 1$. Consequently, the state variable x_t evolves according to:

$$x_{t+1} = \frac{a_{1t}}{a_{2t}x_t + a_{3t}} \cdot x_t, \tag{A.22}$$

where:

$$\begin{aligned}
a_{1t} &= \left[\sigma_t B_t^{\alpha} \left(\rho \frac{\lambda_{Kt}}{\lambda_t}\right)^{\alpha\epsilon} \left(\eta \frac{\lambda_{Lt}}{\lambda_t}\right)^{\alpha(1-\epsilon)} \lambda_{Kt}^{\gamma}\lambda_{Lt}^{1-\gamma} + (1-\delta)\right]^{\epsilon} \\
a_{2t} &= \lambda_t^{\mu} B_t \left(\rho \frac{\lambda_{Kt}}{\lambda_t}\right)^{\epsilon} \left(\eta \frac{\lambda_{Lt}}{\lambda_t}\right)^{1-\epsilon} \\
a_{3t} &= (1-\lambda_t^{\mu})
\end{aligned}$$

A non-stochastic stationary equilibrium for a fixed value of B consists of the variables $(\lambda, \lambda_K, \lambda_L, \theta, p_2, \sigma, x)$ which solve the time-invariant versions of equations: (A.3), (A.6), (A.7), (A.11), (A.19), (A.20), and (A.22).

B. Growth Computations

B.1. Growth

The economy's GNP, measured in terms of output of the $R\&D$ performing producers, is given by.

$$Y_t = \lambda_t p_{1t} c_{1t} + (1-\lambda_t) p_{2t} c_{2t} + \lambda_t k_{t+1}, \tag{B.1}$$

where c_{1t} denotes the consumption of a typical good produced by an $R\&D$-performing producer and c_{2t} is the consumption of a typical good performed in the other sector. Using (A.1), (A.2), and (A.8), GNP can be expressed in terms of the output of a representative $R\&D$-performing producer, y_{1t}:

$$Y_t = y_{1t}\left[\lambda_t \frac{1}{\nu}(1-\sigma_t) + (1-\lambda_t) p_{2t} \frac{1-\sigma_t}{\theta_t} + \lambda_t \sigma_t\right] \tag{B.2}$$

Omitting time indices wherever possible, along a balanced growth path we get:

$$\begin{aligned}
y_{1t} &= \left[B\left(\rho \frac{\lambda_K K_t}{\lambda}\right)^{\epsilon} \left(\eta \frac{\lambda_L}{\lambda}\right)^{1-\epsilon}\right]^{\alpha} \left((1-\rho)\frac{\lambda_K K_t}{\lambda}\right)^{\gamma} \left((1-\eta)\frac{\lambda_L}{\lambda}\right)^{1-\gamma} \qquad \text{(B.3)} \\
&= B^{\alpha}\rho^{1-\gamma}\eta^{\alpha-(1-\gamma)}(1-\rho)^{\gamma}(1-\eta)^{1-\gamma}\lambda_K(\lambda_L)^{\alpha}\lambda^{-\alpha}K_t
\end{aligned}$$

Accordingly, GNP growth rate along a balanced growth path is:

$$\begin{aligned} g &\equiv \frac{y_{1t+1}}{y_{1t}} \\ &= \frac{K_{t+1}}{K_t} = \frac{\lambda_t \sigma_t y_{1t} + (1-\delta) K_t}{K_t} \\ &= \sigma B^{\alpha} \rho^{1-\gamma} \eta^{\alpha-(1-\gamma)} (1-\rho)^{\gamma} (1-\eta)^{1-\gamma} \lambda_K (\lambda_L)^{\alpha} \lambda^{-\alpha} + (1-\delta) \end{aligned} \tag{B.4}$$

B.2. Output shares

We obtain (from (B.1)) that the GNP share of the consumption goods of all $R\&D$-performing producers is:

$$C_1/Y = \frac{\lambda \frac{1}{\nu}(1-\sigma)}{\lambda \frac{1}{\nu}(1-\sigma) + (1-\lambda) p_2 \frac{1-\sigma_1}{\theta} + \lambda\sigma}. \tag{B.5}$$

The GNP share of aggregate consumption of the non-$R\&D$-performing producers is:

$$C_2/Y = \frac{(1-\lambda) p_2 \frac{1-\sigma}{\theta}}{\lambda \frac{1}{\nu}(1-\sigma) + (1-\lambda) p_2 \frac{1-\sigma_1}{\theta} + \lambda\sigma}, \tag{B.6}$$

whereas the (measured) GNP share of aggregate investment is:

$$S/Y = \frac{\lambda\sigma}{\lambda \frac{1}{\nu}(1-\sigma) + (1-\lambda) p_2 \frac{1-\sigma}{\theta} + \lambda\sigma}. \tag{B.7}$$

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