

The Tradeoff of the Commons

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Abstract

We develop a model of scarce, renewable resources to study the commons problem. We show that, contrary to conventional wisdom, property rights can often be less efficient than a commons. In particular, we study two effects: (1) waste which arises when individuals expend resources to use a resource unavailable due to congestion and (2) the risk of underutilization of the resource. We provide necessary and sufficient conditions for each effect to dominate the other when the cost of determining the availability of a resource is low.

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1 Introduction

Many resources are allocated on a first-come, first serve basis. A common example is that of picnic tables in public parks. This “open” arrangement may result in low value use of the picnic tables when high value users are waiting. The waiting could be eliminated through the use of reservations, commonly used to allocate the use of other park facilities, including athletic fields, campsites, and parking lots. Yet few parks allow individuals to reserve picnic tables, park benches, or childrens’ swings.

Open picnic tables involve the organizational form known as a commons. While reservations would assign property rights of a specified time and table to a picnicker, open picnic tables do not. Instead, the resource is opened freely to all. The commons organizational

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form is generally considered a tragedy because it lacks a mechanism to prevent overuse of resources, leading to inefficient allocation (Hardin, 1968).¹ But if it is a tragedy, why is this organizational form so popular among park administrators? We will show here that in many settings, the open arrangement is actually more efficient than that of reservations.²

Scheduling picnic tables in advance will idle resources and may also lead to overuse of the resources, at least when the reservation is not priced. Moreover, at the time of a reservation, the value is not known and may change, resulting in a “jumping the gun” inefficiency of the kind detailed by Roth and Xing (1994). Open picnic tables, in contrast, incur higher waiting costs and may also be inefficiently utilized unless an auction is held for the slot. The important thing to understand is that both advance scheduling and ex post allocation involve inefficiencies, so that the comparison is not at all obvious and that, in particular, the commons approach may dominate. Either can be more efficient, and this paper provides a characterization of efficiency.

We model the distribution of a single good that is costless but available only in a quantity insufficient to meet demand. Scarce natural resources such as wireless spectrum fit this description. Alternatively, our model describes the case where production is costly but cannot be conditioned on realized demand, and where undistributed units of the good lose their value. This would describe, for example, the sale of airline seats, the quantity of which normally cannot be adjusted in the short run. There is a finite set of individuals with single-unit demands for the good, drawn from some known distribution. Recipients of the good pay a fixed price.

There are two dates. At time zero, the individuals may call for reservations, if reservations are allowed. We study both the case where individuals know the realizations of their value at time zero as well as the case where individuals learn of their valuation at time one. At time one, the individuals choose whether to arrive at the distribution center, and incur a transportation cost if they arrive. Holders of reservations are guaranteed a unit of the good, while those without reservations are randomly selected to receive a unit of the good.

¹Each individual who arrives imposes a cost on the others (a lower probability of getting a picnic table) but captures all of the benefits of arriving. The result is that too many picnickers arrive.

²Heller and Eisenberg (1998) also argue that the organizational form of property rights can also lead to a tragedy because of a transaction cost in ex post transfer. If multiple parties have property rights over different aspects of a single resource, the transaction costs involved in assembling the bundle of rights necessary to utilize the resource can lead to underuse. We ignore this specific issue by assuming that property rights are allocated in efficiently sized bundles. The transaction cost in our model is the cost of coordinating which picnickers get to use the picnic tables. We explicitly model this cost through the transportation cost incurred by picnickers who chose to arrive at the park.

The model contains the following tradeoff: without reservations, some individuals may arrive who are not served, leading to a wasted transportation cost. This waste corresponds to overuse in the traditional commons model. The risk of not being served, however, keeps individuals with relatively low demands from arriving, leading to the allocation of the good to higher value users.

We prove several results applicable when the transportation cost is low. Define the *expected surplus* of a distribution as a function which returns, for any price, the expected consumer surplus conditional on trade at that price. For the case where individuals know the realizations of their values at the time that the reservations are made, we show that, if the good is unpriced, then a necessary and sufficient condition for reservations to be optimal is that the expected surplus is decreasing at zero. Furthermore, if prices are positive, then a sufficient condition for reservations to be optimal is that the expected surplus is decreasing at the price. If the welfare-maximizing price is chosen in each regime, then reservations are strictly optimal. For the case where individuals do not know their valuations at time zero, we show that reservations are never optimal.

Early models of the tragedy of the commons were introduced by Gordon (1954) and Scott (1955) in the context of commercial fisheries and similar industries such as hunting and oil production. We note that these resources are exhaustible — a fishery that is overexploited today may not exist tomorrow. In contrast, the resources studied in our model, such as wireless spectrum and airline seats, are perfectly renewable. Unfilled seats on an airline flight today can not be used in the future.

There are many environments that mirror park facilities with respect to the commons. For example, the use of unassigned Wi-Fi and cordless phone spectrum is analogous to open picnic tables. Coase (1959) argued that property rights should be developed over wireless spectrum. While much of this spectrum is closed, there are notable exceptions. Radio spectrum for cordless phones, CB radio, walkie-talkies, and wireless computer connections known as wi-fi (802.11b and g) are open — any complying use is permitted. Manufacturers just use the spectrum as much as they want for complying devices. We notice the commons problem when our computers try to connect to someone else's insecure wireless access point, or when our phone picks up a neighbor's call, but overall the arrangement works well and certainly has led to a proliferation of devices. It is not plausible that assigning property rights to the spectrum would produce higher value than the current arrangement in these applications.

In recent years a significant debate has emerged over the regulation of wireless spectrum. Benkler (1998) and Noam (1998) advocate open spectrum, in part on the grounds that new technologies enable more efficient use of the airwaves so that spectrum is no longer a scarce good. The latter proposed that, in the event of continuing scarcity at peak moments, spectrum should remain open but be priced, a call supported by Benkler (2002). With or without a price, the regime of open spectrum corresponds to the case of open tables.

The proposal for open spectrum was opposed by Hazlett (1998), who argued that spectrum is still scarce, and that a regime of open but priced access would impose prohibitive transaction costs. The argument that continuing scarcity recommends closed spectrum was seconded by Cave and Webb (2004), among others. The regime of property rights corresponds to the case of reservations.

2 The model

There is a finite set of n agents and a scarce indivisible good of which m units are available, $n > m > 0$. Each agent has a demand for a single unit of this good which is drawn i.i.d. from a known distribution F with density f and with support contained in $[0, \infty)$. Reservations may or may not be scheduled in advance. At the distribution time, agents choose to arrive (or not) at the distribution point and incur transportation cost c if they arrive. The m units of the good are then sold to the individuals who arrive at price p , with those individuals who have reservations being given first priority.

2.1 Individual values known at the time of the reservation

In the case of “open tables” there are no reservations. Individuals who arrive are not guaranteed a unit of the good. An individual will choose to arrive if her valuation exceeds a cutoff $v^* \geq p + c$. This gives a binomial distribution of the number of others who arrive, and if i others arrive, the probability of service is $\min\left\{\frac{m}{i+1}, 1\right\}$. Thus the probability of service is

$$\alpha_o \equiv \sum_{i=0}^{n-1} \binom{n-1}{i} (1 - F(v^*))^i F(v^*)^{n-1-i} \min\left\{\frac{m}{i+1}, 1\right\}.$$

The value v^* is defined by the equation $c = (v^* - p) \alpha_o$ as this sets the expected gain equal to the cost for the marginal person. The probability that i people arrive is $\binom{n}{i} (1 - F(v^*))^i F(v^*)^{n-i}$. At most m units of the good can be distributed, thus the expected number of individuals

who are served is given by

$$\kappa(v^*) \equiv \sum_{i=0}^n \binom{n}{i} (1 - F(v^*))^i F(v^*)^{n-i} \min \{m, i\}.$$

The expected value of the good to an individual who arrives is

$$E[v|v \geq v^*] = \int_{v^*}^{\infty} \frac{xf(x)}{1 - F(v^*)} dx.$$

The total social welfare is the expected number of people who are served, times their expected values, minus the expected number of people who arrive, $n(1 - F(v^*))$, times their transportation cost c , or:

$$W_o(c) = \kappa(v^*)E[v|v \geq v^*] - n(1 - F(v^*))c.$$

In the case of “reservations,” individuals are allocated rights to use the tables in advance. Individuals know their valuations at the time of making the reservation. Calling for a reservation is assumed to be costless, so all individuals whose valuations exceed $p + c$ will call for a reservation. The first m callers will be awarded reservations and will be guaranteed a unit of the good. The total social welfare is the expected number of people who arrive times their expected values less their transportation cost, or

$$W_r(c) = \kappa(p + c)(E[v|v \geq p + c] - c).$$

Note that if the transportation cost c is zero, then $v^* = p$, and thus social welfare is the same under both reservations and open tables, that is, $W_r(0) = W_o(0)$.

Define an individual’s expected surplus as the amount by which the individual’s expected value exceeds an amount x , conditional on the value being higher than x , or

$$\Gamma(x) = E[v|v \geq x] - x.$$

We derive a necessary and sufficient condition for when the transportation cost c is small and the good is unpriced. In this case, open tables dominate reservations if and only if the expected surplus is increasing at zero; that is, if $\Gamma'(0) \geq 0$.³

Theorem 1. *For sufficiently small c , if the good is unpriced, $W_o(c) \geq W_r(c)$ if and only if $f(0) \int_0^{\infty} xf(x)dx \geq 1$, or equivalently, $\Gamma'(0) \geq 0$.*

³A similar condition is derived in a different setting by Chakravarty et al. (2010).

For the uniform distribution, reservations dominates open tables. For the exponential distribution with a zero base, the expected surplus is constant, and thus this is a transition case. Finally, if $F(x) = x^a$, for $a < 1$, open tables dominates reservations for sufficiently small costs c . Roughly speaking, open tables dominate reservations if the frequency of low values is very high, more than one over the mean.

Many theoretical studies assume that hazard rates are non-decreasing or, equivalently, that the probability distribution is log-concave. This condition is sufficient to imply that the expected surplus is non-increasing.⁴ In such a case, reservations dominates open tables for any price, not just the zero price contemplated in Theorem 1, as we now show.

Theorem 2. *Suppose that the price $p > 0$ and that $\Gamma'(p) \leq 0$. For sufficiently small c , $W_r(c) > W_o(c)$.*

2.2 Varying prices

Up until this part of the paper, we have assumed that the price p is the same for both open tables and reservations. Alternatively, one might allow these prices to vary. Formally, let p_o be the price chosen under open tables, with $W_o(c, p_o)$ denoting the resulting social welfare, and let p_r be the price chosen under reservations, with $W_r(c, p_r)$ denoting social welfare under reservations. If prices are chosen to maximize social welfare, and the transportation cost is low, then reservations strictly dominates open tables. The proof is left for readers.

Theorem 3. *Suppose that the price $p_o = \arg \max_p W_o(c, p)$, and that the price $p_r = \arg \max_p W_r(c, p)$. For sufficiently small c , $W_r(c, p_r) > W_o(c, p_o)$.*

Alternatively, the price might be set by a profit-maximizing firm, such as a restaurant. In this case, reservations often dominate, but not always. As in the case of Theorem 2.2, a sufficient condition for reservations to dominate open tables is that the expected surplus be decreasing at the price. If open tables dominates, then the price set by the firm is below the socially optimal price. At profit maximizing price, reservations are always optimal for firm.

2.3 Individual valuations unknown at the time of the reservation

In some settings, it may be more realistic to assume that agents do not know their valuations at the time that the reservation is made. In this case, all agents call for reservations which

⁴For more on the relationship between these assumptions, see Bagnoli and Bergstrom (2005). They refer to the expected surplus as the Mean Residual Lifetime Function.

are awarded to the first m callers. Agents with reservations arrive to purchase the unit of the good if their valuations exceed $p + c$. Agents without reservations may choose to arrive hoping to purchase one of the expected $mF(p + c)$ units remaining. These individuals will arrive if they have sufficiently high valuations, exceeding a value $\hat{v} \geq p + c$, and will be served with probability

$$\alpha_s \equiv \sum_{i=0}^m \binom{m}{i} (1 - F(p + c))^i F(p + c)^{m-i} \sum_{j=0}^{n-m-1} \binom{n-m-1}{j} (1 - F(\hat{v}))^j F(\hat{v})^{n-m-1-j} \min \left\{ \frac{m-i}{j+1}, 1 \right\},$$

where \hat{v} is defined by the equation $c = (\hat{v} - p) \alpha_s$. The formulation of α_s can be seen as follows. There are m people with reservations, but only the number i with a value exceeding $p + c$ appear, leaving $m - i$ available slots. Those people without reservations but with values exceeding \hat{v} risk standing in line to get served, and this is also binomially generated.

The total number of individuals without reservations who are served is given by

$$\lambda(\hat{v}, p + c) \equiv \sum_{i=0}^m \binom{m}{i} (1 - F(p + c))^i F(p + c)^{m-i} \sum_{j=0}^{n-m} \binom{n-m}{j} (1 - F(\hat{v}))^j F(\hat{v})^{n-m-j} \min \{m - i, j\}.$$

Total social welfare is given by:

$$W_s(c) = m(1 - F(p + c)) (E[v|v \geq p + c] - c) + \lambda(\hat{v}, p + c) E[v|v \geq \hat{v}] - (n - m)(1 - F(\hat{v})) c.$$

In this case it is less obvious, but equally true, that when the transportation cost c is zero, social welfare is the same under both reservations and open tables.

Lemma 1. $W_o(0) = W_s(0)$.

When the agents do not know their valuations in advance, and the transportation cost c is sufficiently small, more individuals are served under the policy of open tables than under reservations.

Lemma 2. For sufficiently small c , $\kappa(v^*) \geq m(1 - F(p + c)) + \lambda(\hat{v}, p + c)$.

Furthermore, if $\sup\{p|F(p + c) < 1\} > p > 0$, then $\kappa(v^*) > m(1 - F(p + c)) + \lambda(\hat{v}, p + c)$.

This implies that if individuals do not know their values in advance, a profit-maximizing firm will prefer open tables to a policy of reservations.

When the agents do not know their valuations in advance, and the transportation cost c is sufficiently small, open tables dominates reservations regardless of the price chosen and regardless of the distribution of the valuations. If prices are strictly positive, a policy of open tables strictly dominates a policy of reservations. This theorem does not depend on the assumption that the expected surplus is non-increasing.

Theorem 4. *For sufficiently small c , $W_o(c) \geq W_s(c)$ for every distribution F . Furthermore, if $\sup\{p|F(p+c) < 1\} > p > 0$, then $W_o(c) > W_s(c)$.*

3 Conclusion

Should the Federal Communications Commission open some spectrum to all firms, providing unlicensed bands as it did with wi-fi spectrum? Traditional property-rights based analysis suggests not, and several authors have echoed this view. We study this problem through an analogous model of picnic table allocation and show that, while interference is possible, unlicensed bands may lead to more intensive use of the spectrum than would arise under licensing. We prove several results about the optimality of reservations when transportation costs are low.

First, if values are known at the time that reservations are made and the item is unpriced, then reservations are optimal if and only if the expected surplus is decreasing at zero. Roughly speaking, providing unlicensed spectrum is advantageous over licensing when there is a large chance of very low value use. For low power, low interference devices like Wi-Fi routers, this seems quite plausible, while for higher power, high interference devices like cellular phones, licensing seems optimal.

Second, if values are known and the item is priced, then reservations dominates if the expected surplus is decreasing at the price. Third, if values are not known at the time that reservations are made, then reservations are never optimal. This last result is true even though the distribution of values is known at the time reservations are made.

Our results apply when the transportation cost is low. Before transmitting, a Wi-Fi transmitter must implement the “carrier sense multiple access with collision avoidance” protocol.⁵ The cost of implementing this protocol (in terms of time) is small but not insignificant in the aggregate; thus the Wi-Fi example closely fits our assumption of a low transportation cost. Were the transportation cost to become sufficiently large, the probability of congestion would approach zero. In this case, the behavior induced by the two mechanisms converges, and individuals participate if their value is greater than the price plus the transportation cost.

Wi-Fi networks are designed to handle a limited number of connections; this mirrors our assumption that there are m indivisible units of the resource. Once all connections are

⁵See IEEE Std 802.11-2007, Section 9.1.1

being utilized, no more devices may connect to the network. One noticeable distinction between the Wi-Fi example and our model is that the speed of the wireless network may decrease as more devices connect. Whether this detail is significant probably depends on the extent of the slowdown and the uses which are made of the internet connection. It is likely insignificant if the Wi-Fi is used for simple web-browsing and checking e-mail, but could be more relevant if the connection is used for bandwidth intensive applications.

Lessig (2001) argued that unlicensed bands lead to higher rates of growth in telecommunications technology. While we can neither confirm nor reject this hypothesis, our model does suggest the existence of a relationship between the commons and innovation. A high rate of change in the use of the resource will make it harder for individuals to predict their future values. As a result, our last theorem suggests that unlicensed bands are preferable in the presence of innovation. It is not clear whether innovations in telecommunications hardware occur at a fast enough rate to affect values in the manner described in the paper. However, we note that such changes need not involve physical technology — innovations in software and by websites may also affect individuals' values.

The problem studied in our model also emerges in the sale of time-dependent goods and services, such as rental cars, airline seats, and movie tickets. The relationship between private vehicles and membership-based car and bicycle sharing programs, including zipcar, is analogous to the relationship between assignment of property rights and the open tables regime. Ski resorts and amusement parks commonly assign rides on a first-come first-serve basis through queues, and generally do not take reservations for rides at specific times. Barro and Romer (1987) showed that this method is nearly efficient even in times of peak demand. In contrast, art museums, such as the Getty Villa near Los Angeles, regularly take reservations in periods of high demand.

Our theory might also be applied to the distribution of influenza vaccines. Demand for these vaccines varies widely across members of the population according to their ages, health conditions, employment status, and general aversion to being sick. Because individuals generally know their values far in advance, our theory suggests that reservations are likely to be optimal. While the open regime is more common when excess vaccines are available, a combination of reservations (in the form of doctors appointments) and priority rationing (to the elderly) is commonly used to allocate the vaccines in times of scarcity.

We have assumed that the population is finite. This assumption implies that the number of individuals who choose to arrive is a random variable. If we were to model the population

with a non-atomic measure space, the number who arrive would be deterministic, and the main effect of the model would vanish, as there would never be underuse of the resource. Alternatively, one might assume that the size of the population is itself stochastic. We would not expect the results to differ significantly in this case.

We have not discussed the mechanism by which prices are determined, with the exception of Section 2.2 in which prices are chosen to maximize social welfare. While prices must be fixed in advance to be consistent with the commons problem, the actual mechanism used to set prices is not essential to the results. (Theorem 1 applies only when the good is unpriced. In Theorems 2 and 4 dominance is shown regardless of price (in the former case, as long as the probability distribution is log-concave.)) A mechanism that allows prices to be determined based on realized demand will of course be efficient.

There are, of course, many potential mechanisms which can be used to allocate scarce resources. We focus on open tables and reservations because they correspond to the property rights and commons regimes, respectively. We highlight five alternative models worth studying in the future. First, in other contexts, a natural mechanism to study would be the queuing problem, in which individuals line up, and are served in the order in which they arrive. Open tables is a special case of this mechanism. Second, one might also wish to study the more general problem with discounting.

Third, while we have studied the case in which the price can differ between the open tables and reservations regimes (Section 2.2), one might consider the distinct case where the reservations themselves are priced. This alternative case is related to models of congestion studied by Chander and Leruth (1989).⁶ We note, however, that in practice it may not be possible to price the reservations.

Fourth, as argued by Coase (1960), both regimes would be fully efficient were there no transaction costs and were property rights tradable. One might study the model in which agents may trade their reservations, albeit at some cost. (In our model, the distinction between reservations and open tables exists only in the world of positive transaction costs ($c > 0$).) While the cost of trading reservations will be prohibitively high in most cases, trading could be feasible in some scenarios.

Fifth, in our model the resources are excludable and indivisible. Only a fixed number of people (or groups) may use the picnic tables at a given moment in time. In some settings,

⁶We thank the referee for this insight.

however, the quantity of users is not fixed, but the quality of the resource degrades as the number of users increases. An interesting question is whether the insights of this model can be generalized to this alternate case.

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Appendices

A Proof of Theorems 1 and 2.

First, we note that when the transportation cost c is zero, the open tables and reservations models are equivalent; that is, $W_o(0) = W_r(0)$. It follows that for sufficiently small transportation costs c , open tables dominates reservations ($W_o(c) \geq W_r(c)$) when $W'_o(0) \geq W'_r(0)$. By computing these derivatives we find that $W'_o(0) \geq W'_r(0)$ if and only if

$$\kappa(p) \frac{f(p)}{1 - F(p)} (E[v|v \geq p] - p) \geq \kappa(p) - \kappa'(p) E[v|v \geq p]. \quad (1)$$

One can readily compute that $\kappa'(0) = 0$. Thus, after dividing by $\kappa(p)$ and evaluating at $p = 0$, statement (1) reduces to $f(0) \int_0^\infty xf(x)dx \geq 1$, or that the expected surplus is increasing at zero. This proves Theorem 1.

Next, if the expected surplus is non-increasing in the price, or $\Gamma'(p) \leq 0$, then:

$$\frac{f(p)}{1 - F(p)} (E[v|v \geq p] - p) \leq 1. \quad (2)$$

Next, assume that price p is strictly positive and that the expected surplus is non-increasing in the price. Because $\kappa'(p) E[v|v \geq p] < 0$ for all $p > 0$, statement (2) implies that

$$\kappa(p) \frac{f(p)}{1 - F(p)} (E[v|v \geq p] - p) < \kappa(p) - \kappa'(p) E[v|v \geq p].$$

Therefore statement (1) is false. Consequently, $W'_o(0) < W'_r(0)$. This proves Theorem 2.

B Proof of Lemma 1.

If the transportation cost c is zero, then $\hat{v} = p$, and thus $W_s(0) = m(1 - F(p))E[v|v \geq p] + \lambda(p, p)E[v|v \geq p]$. Because $m(1 - F(p)) + \lambda(p, p) = \kappa(p)$, it follows that $W_s(0) = \kappa(p)E[v|v \geq p] = W_o(0)$.

C Proof of Lemma 2.

Let $O(c) = \kappa(v^*)$ be the number served with open tables, and let $S(c) = m(1 - F(p + c)) + \lambda(\hat{v}, p + c)$ be the number served with reservations when individuals do not know their values at time zero. Then $O(0) = \kappa(p)$ and $S(0) = m(1 - F(p)) + \lambda(p, p) = \kappa(p)$. It follows that for sufficiently small transportation costs c , more are served with open tables than with reservations when $O'(0) \geq S'(0)$, and strictly more are served with open tables than with reservations when $O'(0) > S'(0)$. Computing these derivatives, we find that $O'(0) = \frac{n(1-F(p))\kappa'(p)}{\kappa(p)}$, and $S'(0) = \frac{d}{dc}\lambda\{\hat{v}, p + c\}\Big|_{c=0} - mf(p)$. Comparison of these terms shows that the former ($O'(0)$) is larger than the latter ($S'(0)$) for all prices and all distributions, and is strictly larger when p is positive. This proves the lemma.

D Proof of Theorem 4.

By Lemma 1, when the transportation cost c is zero, the open tables and reservations models are equivalent; that is, $W_o(0) = W_s(0)$. It follows that for sufficiently small transportation costs c , open tables weakly dominates reservations ($W_o(c) \geq W_s(c)$) when $W_o'(0) \geq W_s'(0)$, and that open tables strictly dominates reservations ($W_o(c) > W_s(c)$) when $W_o'(0) > W_s'(0)$. By computing these derivatives we find that $W_o'(0) \geq W_s'(0)$ if and only if

$$\frac{n(1-F(p))\kappa'(p)}{\kappa(p)} \geq \frac{d}{dc}\lambda\{\hat{v}, p + c\}\Big|_{c=0} - mf(p). \quad (3)$$

This is the statement that $O'(0) \geq S'(0)$ (see the proof of Lemma 2), and therefore, by Lemma 2, $W_o'(0) \geq W_s'(0)$ for every distribution F . Furthermore, if the price p is strictly positive, then $W_o'(0) > W_s'(0)$. This proves Theorem 4.

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