Unscheduled Appointments

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Abstract

We develop a model of faculty office hours to study the allocation of resources such as wireless spectrum. We show that, contrary to conventional wisdom, property rights can often be less efficient than a commons. In particular, we study two effects: (1) waste which arises when individuals expend resources to use a resource unavailable due to congestion and (2) the risk of underutilization of the resource. We provide necessary and sufficient conditions for each effect dominates the other when the cost of determining the availability of a resource is low.

1 Introduction

Few academics schedule all of their office hours individually with students, preferring to schedule some hours when any students who desire may arrive. This "open" arrangement creates waiting on occasion, and may also result in low-value use of time. As a meeting strategy, it is generally defended on the principle that the office hours are used more intensely, that is, the resource of the professor's time is used more efficiently.

Open office hours involve the organizational form known as a commons. Appointments assign property rights of a specified time to a student, while open office hours do not, opening the resource freely to all. The commons organizational form is generally considered a tragedy because it lacks a mechanism to prevent overuse of resources, leading to inefficient allocation (Hardin, 1968).[1] But if so, why is this organizational

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[1]Each student who arrives imposes a cost on the others (a lower probability of meeting with the professor) but captures all of the benefits of arriving. The result is that too many students arrive.

form so popular among academics, including economists? We will show here that in many settings, the open arrangement is actually more efficient than appointments.[2]

Scheduling appointments in advance will idle resources and may also lead to overuse of the resources, at least when the appointment is not priced. Moreover, at the time of an appointment, the value is not known and may change, resulting in a "jumping the gun" inefficiency of the kind detailed by Roth and Xing (1994). Open office hours, in contrast, incur higher waiting costs and may also be inefficiently utilized unless an auction is held for the slot. The important thing to understand is that both advance scheduling and ex post allocation involve inefficiencies, so that the comparison is not at all obvious and in particular the commons approach may dominate. Either can be more efficient, and this paper provides a characterization of efficiency.

There are many other settings that mirror office appointments with respect to the commons. We will argue below that the use of unassigned Wi-Fi and cordless phone spectrum is analogous to unscheduled appointments. In addition, the relationship between private vehicles and membership-based vehicle sharing programs, such as zipcar and analogous products for bicycles, is analogous to the advance assignment of property rights or not. Indeed, there are a variety of settings where the commons is potentially a superior organizational form to the ex ante assignment of property rights.

We model the distribution of a single good that is costless but available only in a quantity insufficient to meet demand. Scarce natural resources such as wireless spectrum fit this description. Alternatively, our model describes the case where production is costly but cannot be conditioned on realized demand, and where undistributed units of the good lose their value. This would describe, for example, the sale of airline seats, the quantity of which normally cannot be adjusted in the short run. There is a finite set of individuals with single-unit demands for the good, drawn from some known distribution. Recipients of the good pay a fixed price.

There are two dates. At time zero, the individuals may call for reservations, if reservations are allowed. We study both the case where individuals know the realizations of their value at time zero as well as the case where individuals learn of their valuation at time one. At time one, the individuals choose whether to arrive at the distribution center, and incur a transportation cost if they arrive. Holders of reservations are

[2]Heller and Eisenberg (1998) also argue that the organizational form of property rights can also lead to a tragedy because of a transaction cost in ex post transfer. If multiple parties have property rights over different aspects of a single resource, the transaction costs involved in assembling the bundle of rights necessary to utilize the resource can lead to underuse. We ignore this specific issue by assuming that property rights are allocated in efficiently sized bundles. The transaction cost in our model is the cost of coordinating which students attend the faculty member's office hours. We explicitly model this cost through the transportation cost incurred by students who chose to arrive at the coordination point (the hallway near the office door).

guaranteed a unit of the good, while those without reservations are randomly selected to receive a unit of the good.

The model contains the following tradeoff: without reservations, some individuals may arrive who are not served, leading to a wasted transportation cost. This waste corresponds to overuse in the traditional commons model. The risk of not being served, however, keeps individuals with relatively low demands from arriving, leading to the allocation of the good to higher value users.

We prove several results applicable when the transportation cost is low. Define the *expected surplus* of a distribution as a function which returns, for any price, the expected consumer surplus conditional on trade at that price. For the case where individuals know the realizations of their values at the time that the reservations are made, we show that, if the good is unpriced, then a necessary and sufficient condition for reservations to be optimal is that the expected surplus is decreasing at zero. Furthermore, if prices are positive, then a sufficient condition for reservations to be optimal is that the expected surplus is decreasing at the price. For the case where individuals do not know their valuations at time zero, we show that reservations are never optimal.

This question has widespread applicability in other commons environments. Coase (1959) argued that property rights should be developed over wireless spectrum. While much of this spectrum is closed, there are notable exceptions. Radio spectrum for cordless phones, CB radio, walkie-talkies, and wireless computer connections known as wi-fi (802.11b and g) are open — any complying use is permitted. Manufacturers just use the spectrum as much as they want for complying devices. We notice the commons problem when our computers try to connect to someone else's insecure wireless access point, or when our phone picks up a neighbor's call, but overall the arrangement works well and certainly has led to a proliferation of devices. It is not plausible that assigning property rights to the spectrum would produce higher value than the current arrangement in these applications.

In recent years a significant debate has emerged over the regulation of wireless spectrum. Benkler (1998) and Noam (1998) advocate open spectrum, in part on the grounds that new technologies enable more efficient use of the airwaves so that spectrum is no longer a scarce good. The latter proposed that, in the event of continuing scarcity at peak moments, spectrum should remain open but be priced, a call supported by Benkler (2002). With or without a price, the regime of open spectrum corresponds to the case of unscheduled appointments.

The proposal for open spectrum was opposed by Hazlett (1998), who argued that spectrum is still scarce, and that a regime of open but priced access would impose prohibitive transaction costs. The argument that continuing scarcity recommends closed spectrum was seconded by Cave and Webb (2004), among others. The regime of property rights corresponds to the case of scheduled appointments.

Early models of the tragedy of the commons were introduced by Gordon (1954) and Scott (1955) in the context of commercial fisheries and similar industries such as hunting and oil production. We note that these resources are exhaustible — a fishery that is overexploited today may not exist tomorrow. In contrast, the resources studied in our model, such as wireless spectrum and airline seats, are perfectly renewable. Unfilled seats on an airline flight today can not be used in the future.

The problem studied in our model also emerges in the sale of time-dependent goods and services, such as rental cars, airline seats, and movie tickets. Ski resorts and amusement parks commonly assign rides on a first-come first-serve basis through queues, and generally do not take reservations for rides at specific times. Barro and Romer (1987) showed that this method is nearly efficient even in times of peak demand. In contrast, art museums, such as the Getty Villa near Los Angeles, regularly take reservations in periods of high demand.

2 The model

There is a finite set of n agents and a scarce indivisible good of which m units are available, $n > m > 0$. Each agent has a demand for a single unit of this good which is drawn i.i.d. from a known distribution F with density f and with support contained in $[0, \infty)$. Reservations may or may not be scheduled in advance. At the distribution time, agents choose to arrive (or not) at the distribution point and incur transportation cost c if they arrive. The m units of the good are then sold to the individuals who arrive at price p, with those individuals who have reservations being given first priority.

2.1 Individual values known at the time of the reservation

In the case of "unscheduled appointments" there are no reservations. Individuals who arrive are not guaranteed a unit of the good. An individual will choose to arrive if her valuation exceeds a cutoff $v^* \geq p + c$. This gives a binomial distribution of the number of others who arrive, and if i others arrive, the probability of service is $\min\left\{\frac{m}{i+1}, 1\right\}$. Thus the probability of service is

$$\alpha_u \equiv \sum_{i=0}^{n-1} \binom{n-1}{i} (1 - F(v^*))^i \, F(v^*)^{n-1-i} \min\left\{\frac{m}{i+1}, 1\right\}.$$

The value v^* is defined by the equation $c = (v^* - p)\,\alpha_u$ as this sets the expected gain equal to the cost for the marginal person. The probability that i people arrive is $\binom{n}{i}(1 - F(v^*))^i F(v^*)^{n-i}$. At most m units of the good can be distributed, thus the expected number of individuals who are served is given by

$$\kappa(v^*) \equiv \sum_{i=0}^{n} \binom{n}{i} (1 - F(v^*))^i F(v^*)^{n-i} \min\{m, i\}.$$

The expected appointment value of an individual who arrives is

$$E[v|v \geq v^*] = \int_{v^*}^{\infty} \frac{xf(x)}{1 - F(v^*)} dx.$$

The total social welfare is the expected number of people who are served, times their expected values, minus the expected number of people who arrive, $n(1 - F(v^*))$, times their transportation cost c, or:

$$W_U(c) = \kappa(v^*)E[v|v \geq v^*] - n(1 - F(v^*))\,c.$$

In the case of "scheduled appointments" reservations are allowed. Individuals know their valuations at the time of making the reservation. Calling for a reservation is assumed to be costless, so all individuals whose valuations exceed $p + c$ will call for a reservation. The first m callers will be awarded reservations and will be guaranteed a unit of the good. The total social welfare is the expected number of people who arrive times their expected appointment values less their transportation cost, or

$$W_A(c) = \kappa(p + c)\,(E[v|v \geq p + c] - c).$$

Note that if the transportation cost c is zero, then $v^* = p$, and thus social welfare is the same under both scheduled and unscheduled appointments, that is, $W_A(0) = W_U(0)$.

Define individual's expected surplus is the amount by which the individual's expected value exceeds an amount x, conditional on the value being higher than x, or

$$\Gamma(x) = E[v|v \geq x] - x.$$

We derive a necessary and sufficient condition for when the transportation cost c is small and the good is unpriced. In this case, unscheduled appointments dominate scheduled appointments if and only if the expected surplus is increasing at zero; that is, if $\Gamma'(0) \geq 0$.

Theorem 2.1. *For sufficiently small c, if the good is unpriced, $W_U(c) \geq W_A(c)$ if and only if $f(0)\int_0^{\infty} xf(x)dx \geq 1$, or equivalently, $\Gamma'(0) \geq 0$.*

For the uniform distribution, scheduled appointments dominates unscheduled appointments. For the exponential distribution with a zero base, the expected surplus is constant, and thus this is a transition case. Finally, if $F(x) = x^a$, for $a < 1$, unscheduled appointments dominates scheduled appointments for sufficiently small costs c. Roughly speaking, unscheduled appointments dominate scheduled appointments if the frequency of low values is very high, more than one over the mean.

Many theoretical studies assume that hazard rates are non-decreasing or, equivalently, that the probability distribution is log-concave. This condition is sufficient to imply that the expected surplus is non-increasing.[3] In such a case, scheduled appointments dominates unscheduled appointments for any price, not just the zero price contemplated in Theorem 2.1, as we now show.

Theorem 2.2. *Suppose that the price $p > 0$ and that $\Gamma'(p) \leq 0$. For sufficiently small c, $W_A(c) > W_U(c)$.*

2.2 Individual valuations unknown at the time of the reservation

In some settings, it may be more realistic to assume that agents do not know their valuations at the time that the reservation is made. In this case, all agents call for reservations which are awarded to the first m callers. Agents with reservations arrive to purchase the unit of the good if their valuations exceed $p + c$. Agents without reservations may choose to arrive hoping to purchase one of the expected $mF(p + c)$ units remaining. These individuals will arrive if they have sufficiently high valuations, exceeding a value $\hat{v} \geq p + c$, and will be served with probability

$$\alpha_s \equiv \sum_{i=0}^{m} \binom{m}{i} (1 - F(p + c))^i \, F(p+c)^{m-i} \sum_{j=0}^{n-m-1} \binom{n - m - 1}{j} (1 - F(\hat{v}))^j \, F(\hat{v})^{n-m-1-j} \min \left\{ \frac{m - i}{j + 1}, 1 \right\}$$

where $\hat{v}$ is defined by the equation $c = (\hat{v} - p)\, \alpha_s$. The formulation of α_s can be seen as follows. There are m people with reservations, but only the number i with a value exceeding $p + c$ appear, leaving $m - i$ available slots. Those people without reservations but with values exceeding $\hat{v}$ risk standing in line to get served, and this is also binomially generated.

The total number of individuals without reservations who are served is given by

$$\lambda(\hat{v}, p{+}c) \equiv \sum_{i=0}^{m} \binom{m}{i} (1 - F(p + c))^i \, F(p{+}c)^{m-i} \sum_{j=0}^{n-m} \binom{n - m}{j} (1 - F(\hat{v}))^j \, F(\hat{v})^{n-m-j} \min \{m - i, j\} .$$

[3]For more on the relationship between these assumptions, see Bagnoli and Bergstrom (2005). They refer to the expected surplus as the Mean Residual Lifetime Function.

Total social welfare is given by:

$$W_S(c) = m\,(1 - F(p + c))\,(E\,[v|v \geq p + c] - c) + \lambda(\hat{v}, p+c)E\,[v|v \geq \hat{v}] - (n-m)\,(1 - F(\hat{v}))\,c.$$

In this case it is less obvious, but equally true, that when the transportation cost c is zero, social welfare is the same under both scheduled and unscheduled appointments.

Lemma 2.3. $W_U(0) = W_S(0)$.

When the agents do not know their valuations in advance, and the transportation cost c is sufficiently small, unscheduled appointments dominates scheduled appointments regardless of the price chosen and regardless of the distribution of the valuations. If prices are strictly positive, a policy of unscheduled appointments strictly dominates a policy of scheduled appointments. This theorem does not depend on the assumption that the expected surplus is non-increasing.

Theorem 2.4. *For sufficiently small c, $W_U(c) \geq W_S(c)$ for every distribution F. Furthermore, if* $\sup\{p|F(p + c) < 1\} > p > 0$*, then* $W_U(c) > W_S(c)$.

3 Conclusion

Should the Federal Communications Commission open some spectrum to all firms, providing unlicensed bands as it did with wi-fi spectrum? Traditional property-rights based analysis suggests not, and several authors have echoed this view. We study this problem through an analogous model of appointments and show that, while interference is possible, unlicensed bands may lead to more intensive use of the spectrum than would arise under licensing. We prove several results about the optimality of scheduling when transportation costs are low.

First, if values are known at the time that reservations are made and the item is unpriced, then scheduling is optimal if and only if the expected surplus is decreasing at zero. Roughly speaking, providing unlicensed spectrum is advantageous over licensing when there is a large chance of very low value use. For low power, low interference devices like Wi-Fi routers, this seems quite plausible, while for higher power, high interference devices like cellular phones, licensing seems optimal.

Second, if values are known and the item is priced, then scheduling dominates if the expected surplus is decreasing at the price. Third, if values are not known at the time that reservations are made, scheduling is never optimal. This last result is true even though the distribution of values is known at the time reservations are made.

Lessig (2001) argued that unlicensed bands lead to higher rates of growth in telecommunications technology. While we can neither confirm nor reject this hypothesis, our model does suggest the existence of a relationship between the commons and innovation. A high rate of change in the use of the resource will make it harder for individuals to predict their future values. As a result, our last theorem suggests that unlicensed bands are preferable in the presence of innovation. It is not clear whether innovations in telecommunications hardware occur at a fast enough rate to affect values in the manner described in the paper. However, we note that such changes need not involve physical technology — innovations in software and by websites may also affect individuals' values.

We have assumed that the population of students is finite. This assumption implies that the number of students who choose to arrive is a random variable. If we were to model students with a non-atomic measure space, the number who arrive would be deterministic, and the main effect of the model would vanish, as there would never be underuse of the resource. Alternatively, one might assume that the number of students is itself stochastic. We would not expect the results to differ significantly in this case.

There are, of course, many potential mechanisms which can be used to allocate scarce resources. We focus on scheduled and unscheduled appointments because they correspond to the property rights and commons regimes, respectively. In other contexts, a natural mechanism to study would be the queuing problem, in which individuals line up, and are served in the order in which they arrive. Unscheduled appointments is a special case of this mechanism. The more general problem with discounting is worth studying in future work.

Appendices

A Proof of Theorems 2.1 and 2.2.

First, we note that when the transportation cost c is zero, the unscheduled and scheduled models are equivalent; that is, $W_U(0) = W_A(0)$. It follows that for sufficiently small transportation costs c, unscheduled dominates scheduled ($W_U(c) \geq W_A(c)$) when $W_U'(0) \geq W_A'(0)$. By computing these derivatives we find that $W_U'(0) \geq W_A'(0)$ if and only if

$$\kappa(p)\frac{f(p)}{1-F(p)}\left(E\left[v|v\geq p\right]-p\right)\geq \kappa(p)-\kappa'(p)E\left[v|v\geq p\right]. \tag{1}$$

After dividing by $\kappa(p)$ and evaluating at $p=0$, statement (1) reduces to $f(0)\int_0^\infty xf(x)dx \geq 1$, or that the expected surplus is increasing at zero. This proves Theorem 2.1.

Next, assume that price p is strictly positive and that the expected surplus is non-increasing in the price, or $\Gamma'(p) \leq 0$. Together these assumptions imply that:

$$\kappa(p)\frac{f(p)}{1-F(p)}\left(E\left[v|v \geq p\right]-p\right) < \kappa(p) - \kappa'(p)E\left[v|v \geq p\right],$$

and therefore that $W_U'(0) < W_A'(0)$. This proves Theorem 2.2.

B Proof of Lemma 2.3.

If the transportation cost c is zero, then $\hat{v} = p$, and thus $W_S(0) = m\left(1 - F(p)\right) E[v|v \geq p] + \lambda(p,p)E[v|v \geq p]$. Because $m\left(1-F(p)\right) + \lambda(p,p) = \kappa(p)$, it follows that $W_S(0) = \kappa(p)E[v|v \geq p] = W_U(0)$.

C Proof of Theorem 2.4.

By Lemma 2.3, when the transportation cost c is zero, the unscheduled and scheduled models are equivalent; that is,$W_U(0) = W_S(0)$. It follows that for sufficiently small transportation costs c, unscheduled weakly dominates scheduled ($W_U(c) \geq W_S(c)$) when $W_U'(0) \geq W_S'(0)$, and that unscheduled strictly dominates scheduled ($W_U(c) > W_S(c)$) when $W_U'(0) > W_S'(0)$. For every distribution F, $W_U'(0) \geq W_S'(0)$. Furthermore, if the price p is strictly positive, then $W_U'(0) > W_S'(0)$. This proves Theorem 2.4.

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