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Quantitative growth effects of subsidies in a search theoretic R&D model

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Abstract. Should government subsidize R&D and does it matter how these subsidies are allocated? We examine these questions in a dynamic model where R&D is described as sequential sampling from a distribution of new ideas. Successful discoveries affect future available resources and incentives for further R&D. Consequently, there may be under-investment in R&D. We study the effect of government interventions aimed at fostering growth through R&D. Calibrating the model with aggregate data from the Israeli business sector allows us to quantitatively compare two forms of support resembling those actually used to encourage R&D in the Israeli business sector: (i) an unrestricted subsidy that may be used at the recipients' discretion to finance R&D or other investments, (ii) a subsidy earmarked by the government for R&D activities only. While there is no theoretical way to determine which of the two subsidies will have a greater impact on search for new ideas and growth, we find that in the calibrated economy both subsidies have a significant but similar impact on the economy's output and TFP growth rates. Accordingly, in the case of the Israeli business sector, the incentives to conduct R&D were sufficiently strong, and no R&D-specific encouragement was needed. However, a sensitivity analysis reveals that for economies characterized by other parameter values this result may not be true.

Key words: Subsidies – R&D – Endogenous growth

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1 Introduction

1.1 R&D subsidies

This paper assesses the growth effects of research and development activities and of policies aimed at supporting them. It demonstrates that an endogenous R&D based growth model can be successfully calibrated to mimic the performance of an actual economy (Israel) and exploits this success to numerically examine the growth effects of different subsidy schemes. In particular, it shows that given the parameters induced by the calibration, the R&D stimulus provided by a general subsidy to capital formation is very similar to the stimulus created by an equal subsidy restricted to R&D activities only. This result implies that to the extent that private investment in R&D in the economy examined here is too low, it is because of lack of resources, rather than insufficient incentives to conduct R&D.

There is considerable disagreement in the literature about whether policy can impact long run growth at all, and if so – what kind of policy.[1] Despite this theoretical debate, there is a general sense that governments do have a role in promoting R&D. This role is usually associated with the apparent externalities involved. When private inventors cannot appropriate all the benefits associated with their inventions, private returns to R&D fall short of the social returns. Indeed, some recent attempts at estimating the social returns come up with very high numbers.[2] Under such circumstances it seems obvious that government intervention in the form of subsidies is called for.[3] However, the precise form this intervention should take is unclear. In practice, many governments are actively trying to promote R&D by various forms of subsidies. In some cases the subsidies are very general in scope, while in others they take the restrictive form of grants associated with specific R&D activities.[4]

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[1] In Solow type models growth is determined by exogenous parameters which are unaffected by policy. In some endogenous growth models, (e.g. Grossman and Helpman, 1991; Aghion and Howitt, 1992), policies may have an impact on growth through their effect on knowledge generating activities. Other endogenous growth models require the exogenous growth of a non-reproducible factor of production (Jones, 1995; Kortum, 1997). In these models the steady-state growth rate is policy invariant, as in the Solow model.

[2] For instance, Griliches (1992) found total rates of return on R&D of magnitudes up to 110%, Coe and Helpman (1995) estimated these returns for the $G7$ countries at up to 121.9%. On the other hand, see Jones and Williams (1995) for an evaluation of some of the forces generating excessive R&D investment.

[3] Total subsidies and fiscal incentives to business enterprise R&D acticities in the OECD countries amounted in 1986 to about 0.4% of the combined total output of these countries, but that share has significantly declined since then. In some of the OECD countries, tax deductions for qualifying R&D activities can exceed 100% of the R&D expenditure, (150% in Australia, and 125% in Denmark). See Science and Technology, 1994, OECD, Paris.

[4] Another contentious issue in R&D support policies is the displacement effect of R&D subsidies on firms' own-financed R&D activities. See Lach (2001) for an interesting empirical attempt to estimate the overall impact of the R&D grants administered in Israel by the Chief Scientist of the Ministry of Industry and Trade on privately financed R&D activities.

1.2 The Israeli experience

The Israeli experience with public support to industrial R&D illustrates the potential and pitfalls of such policy measures. Recognizing early on the need to rely on innovative technologies in the absence of industrial base or natural resources, Israeli governments have developed since the early 1970's an extensive entitlement-like system of government support to industrial R&D, which turned into a law in 1988.[5]

The average share of Israeli public support to business R&D over the years in Israel has been about 20%. Two main characteristics of the Israeli R&D support program are its *comprehensiveness* and *intentional neutrality*. Any industrial R&D project satisfying some general feasibility and local production capabilities is entitled to public funding, ranging from 20% to 66% of the R&D costs. The exact subsidy depends on whether the project develops a new product or process or improves an existing one, and whether it is intended for commercial or military purposes.[6] An elaborate oversight and reporting mechanism was established by the major funding government agency, the Office of the Chief Scientist at the Ministry of Industry and Trade, to guarantee that publicly funded support is indeed used for R&D activities. Despite these hurdles, and the fact that this public support entailed some strings in the form of royalties payback on successful projects and restrictions on exports of the knowledge developed with public funding, the majority of R&D projects in the Israeli business sector enjoyed these R&D subsidies over the last 30 years.[7]

In addition to the R&D support program, the Israeli government also provided more general capital formation subsidies to industrial firms, based primarily on their geographic location within the country, and the employment potential of the project to be funded. This type of general capital subsidy has been significantly reduced over the years. The current budget expenditure on this subsidy is about $200 millions, (about half the annual R&D support), and the subsidy rate as a fraction of the qualifying investments is 10–20%, depending on the geographical location of the investment. In its hey days, the annual budget of this capital subsidy was about $600 millions, and the corresponding subsidy rates were higher too – about 30%. In part, the reduction in this type of capital investment subsidy also reflects the much lower level of qualifying investments, primarily in terms of geographic dispersion.[8] Thus there is almost a complete turnabout in the composition of supported industrial

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[5] Between 1969 and 1987, industrial R&D expenditures grew at an average of 14% per year and high-tech exports grew at average annual rate of 12%.

[6] For a more comprehensive description of the extent and development of R&D support programs in Israel see Trajtenberg (2000).

[7] The Israeli R&D support program is currently undergoing a major revision, in part due to the fact that the subsidies implied by its current structure exceeded available resources with the unprecedented growth of R&D in the high-tech sector during the last decade. The revision was prompted by a large $1.8 billions investment by Intel in 1999, most of it in R&D and production facilites. This investment was entitled to government cost-sharing of some 30% at a time when the entire support program budget was about $400 millions.

[8] The sharp reduction in subsidies under the "Law for Encouragement of Capital Investment" is a result of changed government policy as of 1999. Further details are available at the Israeli Ministry of Finance URL: www.mof.gov.il.

investment activities in Israel over a 30 years period, from mostly capital formation subsidies to R&D subsidies, with roughly the same overall levels of support.

The phenomenal growth of the Israeli high-tech sector in the presence of these kinds of government support programs motivates the attempt in this paper to evaluate the overall contribution of subsidies to economic growth, and the impact of earmarking them to R&D activities.

1.3 Restricted vs. unrestricted subsidies

Our analysis of the role of the government in promoting R&D is based on a search theoretic mechanism of endogenous growth, which we explored in Bental and Peled (1996). The model views the process of research and development as a sequential search over a distribution of potential "untried" technologies for an improved production method.[9] The search for better technologies consumes resources which can be used instead to expand firm's operation. Profit maximizing competitive firms determine how to allocate these resources between such two alternative uses. Concurrent with evidence, as technology advances, the search for better technologies requires an ever increasing investment in R&D. This feature generates a potential feedback mechanism, whereby successful investments in R&D lead to technological improvements and higher output which, in turn, provides the resource base from which the increased investments needed to conduct R&D can be financed.

There is no guarantee that this feedback mechanism will be operative. Absent technological improvements, the economy would eventually stop growing since the production process of goods is characterized by the usual decreasing returns to accumulable resources. Therefore, growth can be sustained only if the rate at which the economy's resource base increases (through successful R&D) is higher than the rate at which the input required to generate new technologies increases. Under certain assumptions on the distribution of potential technological improvements, this will be the case.

The rationale for government intervention in the model is provided by the positive R&D externalities it generates. Knowledge created by R&D is assumed to become public very rapidly. However, firms engaged in R&D fail to take this dynamic externality into account, and consequently self-financed R&D investment may be too low.

As the model associates the growth-through-R&D process with the accumulation of wealth, it allows us to examine the growth implications of various government intervention schemes in the capital market. In particular, we are interested in comparing schemes which explicitly support R&D activities to others in which the government provides subsidies to capital formation in general. A general subsidy will fail to have long-run growth effects absent investment in R&D. It would simply shift the economy to another no-growth steady state, possibly with a higher capital/labor ratio. However, if the private sector allocates some of the additional resources it obtains through the subsidy to R&D, a general investment subsidy may

[9] The idea of modeling R&D as sequential search for better technologies has been suggested in various forms by Evenson and Kislev (1975), Nelson and Winter (1982), and Telser (1982).

also have growth effects. Thus, the processes of capital accumulation and R&D investment feed on each other in this model. As capital accumulation increases, holding technology level fixed, the value of technological improvement increases and that of additional capital accumulation decreases. Likewise, following a technological improvement, the value of expanding production capacity which exploits this new technology increases relative to that of further R&D investment. Consequently, any policy aimed at encouraging one of these activities also has an indirect impact on the other. This complicates the analytical comparison of the growth impact of the two subsidies and requires a numerical evaluation which obviously depends on parameter values.

All subsidies are financed by otherwise non-distorting lump-sum taxes. These taxes reduce disposable income and private saving. However, national saving increases as the marginal propensity to save is smaller than unity. Therefore, tax-subsidy schemes of the kind considered here amount to a forced saving program. Compared to the unrestricted subsidy scheme, the R&D-earmarked subsidy is likely to have a larger impact on R&D activities, and increase the probability of finding an improved technology. However, more resources are devoted, on average, to actual production and capital formation under the first scheme. Therefore, growth may be more significantly affected under the first scheme, at least during periods in which it is more profitable to invest in physical capital rather than in R&D.

The effects of the government policies are derived numerically in a model economy whose parameters are chosen to mimic some key features of the Israeli business sector. In particular, the parameters are chosen so as to match the means of the model's analogues of output, labor and capital levels in the 1975–90 period to the actual data. The calibrated model performs rather well, and generates moments which resemble features of the data that were not used in the calibration. Specifically, the total factor productivity growth, the R&D share of output of the business sector, and the capital-output ratios are quite close to their data counterparts.

With the amount of transfers set to about 2% of output, we obtain that the two subsidy schemes have very similar effects on the economy. Both increase the average annual growth rate by about 0.5 percentage points, compared to the rate which would have materialized absent any intervention. Moreover, both policies significantly increase total factor productivity growth (by about 0.15 percentage points). Accordingly, about one third of the gain in the growth rate results directly from the improved technologies. The other two thirds of the sustained growth effect of subsidies are due to the constant increase in the rate of growth of production capital. The similarity in the allocation of resources to R&D and production activities under the two schemes indicates that there has been no need to design specific R&D oriented policies in Israel.

Counterfactual experiments with parameter values show quite clearly that the result is sensitive to parameter choices. Therefore, while we may conclude that the Israeli business sector seems to have had sufficient incentives to allocate resources to R&D on its own, this need not be true in other economic environments.

2 An overview of the model

2.1 Saving and investment

We consider a simple discrete time variant of the Solow growth model. Every person in the population is endowed each period with one unit of labor which is inelastically supplied. For simplicity we assume that saving is a fixed proportion out of disposable income. Having (trivially) determined the overall level of savings, consumers still face the portfolio composition problem of how to allocate their investments among competing firms in the economy. These firms compete with each other not only in the product market, but also for the savings of these consumers, which provide them with resources for capital and R&D investments. In order to succeed in attracting savers' resources, firms attempt to maximize profits, or returns on investment. The full treatment of this problem is very complicated, and involves among other things solving a multi-period, multi-firm dynamic investment problem. We circumvent this difficulty by assuming that all firms are ex-ante identical each period.[10] A further simplification is achieved through the assumption that the the entire business sector can be represented by a single, competitive firm.[11]

2.2 Production and R&D

At each period the representative firm generates profits by engaging in two distinct but related activities: (i) at the beginning of each period, the firm may conduct a costly *sequential search* for a new "technology", with search costs financed by resources raised from the previous period savers;[12] (ii) The remaining resources (net of search costs) are added to the existing capital stock, and are combined with the optimally hired labor input to produce output in a constant returns to scale production technology. The firm incorporates the "best available practice" in its search decision process. This evolving "technological fallback option" is described as the technology that has been used during the previous period, which is always available to the firm at no cost. Profits are returned to the economy, and constitute part of the population's income.

This simplified setup draws its properties from a multi-firm version of the model. In that environment many identical competitive firms independently conduct R&D aimed at increasing their expected profit. As a result of this R&D process, any individual firm may find a profitable technology which it immediately implements. The technology found by each firm is private information during the current period, but becomes publicly known at the end of the period. Consequently, the best

[10] This requires not only the complete diffusion of technology among firms at the end of each period, but also the reallocation of capital among them, so that they face the same research-investment problem at the beginning of the following period. The latter feature is actually a consequence of the first, if investors can perfectly divesrsify their portfolios.

[11] The key implication of this assumption is that the firm is treated as a price-taker in the product and factors markets. See Bental and Peled (1996) for a complete description of this equilibrium.

[12] Specifying R&D costs in terms of output only is a simplifying assumption. What is essential for the model is that an accumulable resource can be used to finance R&D activities.

technology found by searching firms this period will become the technological default available to all firms next period. As no firm knows whose technology will be best, every individual firm associates a very low probability to the event that the technology it discovers this period will also be in use next period and practically ignores the dynamic implications of its current R&D decision. In fact, there is of course a tight link between current R&D activities, and the default technology that is commonly known in the subsequent period. The fact that this link is ignored is the essence of the externality in this environment, which underlies the potential usefulness of government intervention.

2.3 Optimal R&D investment

The firm conducts its search by taking random draws from an infinitely large and unchanging population of "untried" technologies. The firm examines random draws from that population sequentially, incurring a fixed sampling cost per draw paid out of the beginning of period resources. There is no time-cost involved in R&D efforts within the period.

A technology draw completely reveals its productivity level, and the sampling firm can then decide whether to adopt or reject it. Adopting a technology means stopping the search for that period, and investing all remaining resources in production employing that technology. Rejecting a sampled technology means taking at least one more draw. In addition to having at hand the most recently sampled technology, the firm can adopt at any point during the current period the available technological fallback option, and avoid any further search costs.

A search strategy of the firm at any period specifies the rule by which the firm decides when to stop the process of sequential sampling, given the installed production capital, the remaining level of resources that can be used to finance continued search, and the available technologies at hand. The optimal search strategy is characterized by threshold acceptance levels, such that a technology sampled at any stage of the search process is accepted if it exceeds the relevant acceptance threshold, and rejected otherwise. As the amount of resources that can be allocated to R&D declines, the acceptance thresholds decrease, and the firm becomes less fastidious (see Appendix A).

2.4 Government policies

We consider two intervention schemes implemented by the government, both involving a tax-financed transfer of resources from the population to the business sector. Consequently, both schemes amount to a forced increase in savings relative to the intervention-free regime and are likely to increase the economy's growth rate. However, the two schemes work through different channels, and therefore may have different effects on the economy.

The first scheme lets the firm decide how to use the resources transferred to it by the government. In particular, the firm adds this transfer to the resources it obtains directly from the population (implicitly through a capital market), and conducts search and production activities as described above. In the second scheme the government "earmarks" the funds it transfers to the firm. The firm is obliged to use the transferred amount to conduct R&D, (search), and may not use it for production.

If the firm could simply substitute the government transfer for the amount it would have spent on search from its own resources, the two policies would have been equivalent. However, due to the uncertain and sequential nature of the search process, there is no sense in which *ex-ante* substitution can take place. Moreover, given a sufficiently productive default technology, the firm will choose not to conduct any R&D if it may use its resources (including the subsidy) at its discretion. On the other hand, if the transfer may be used only for R&D, the firm will obviously use it for that purpose rather than give the transfer up. Accordingly, an R&D-restricted subsidy may seem relevant in those cases where there is a *prima-facie* presumption that firms would otherwise invest "too little" in R&D, due to their failure to take the dynamic externalities into account.

2.5 Dynamic evolution of the economy

The amount of private saving which is available to the firm every period depends on last period's disposable income, and is predetermined. The amount of the government subsidy (and tax) is also predetermined (in a way to be described below). Pre-search production capital is the amount of production capital of the previous period, which is considered to be "installed" capital, minus depreciation. "New" capital, consisting of savings and possibly government subsidies, provides the resource which the firm may use for search or for further capital formation. At the end of the search phase, production takes place, and the income generated by the firm is channeled back to the economy, in the form of wages and profits. The tax which finances the transfer is paid out of that income. In addition, there are some exogenous leakages (representing the remaining government activities as well as the foreign trade sector), which affect the disposable income. The population saves a fixed proportion out of the current period disposable income. Part of the saving is exogenously allocated to non-industrial investments (such as housing). The remainder becomes available to the business sector, to be used by it next period.

3 A completely specified economy

3.1 The structure of the economy

Population grows exogenously at a rate denoted by x_L. Every agent supplies one unit of labor inelastically per period, so that labor supply, L_t, also grows at the rate x_L,[13]

$$L_t = (1 + x_L)L_{t-1}. \tag{3.1}$$

[13] Specifying exogenous growth in the labor force is done for calibration purposes only, and has no role in generating per-capita sustained growth in this model.

Total output at time t is given by:

$$Y_t = A\theta_t K_t^{\gamma} L_t^{1-\gamma}, \quad A > 0, \quad 0 < \gamma < 1, \quad \theta > 1, \tag{3.2}$$

where θ_t is the index of the technology actually employed at t, and K_t and L_t are, respectively, the capital and labor employed in production at period t.[14]

We define disposable income as:

$$Y_t^D = Y_t + (M_t - X_t) - G_t, \tag{3.3}$$

where Y_t is total income, $M_t - X_t$ is net imports, and government purchases are G_t.[15]

Letting S_t denote total savings at time $t-1$, we have:

$$S_t = \beta Y_{t-1}^D, \quad 0 < \beta < 1. \tag{3.4}$$

In a standard capital accumulation model, S_t is added to the undepreciated amount of capital left from $t-1$, to form the capital stock at time t. Here S_t is the amount of resources to be allocated to non-industrial investments, production of goods, and search for better technologies, (R&D). We denote by IX_t the non-industrial investment at time t, and regard it as exogenously determined. The amount of resources available to the business sector for R&D and production purposes, denoted by Q_t, is then given by:

$$Q_t = S_t - IX_t. \tag{3.5}$$

Out of Q_t, the business sector invests R_t in R&D, a random amount to be endogenously determined during the search process. The rest is added to the stock of production capital. Thus, the law of motion of the stock of *production* capital, K_t, is:

$$K_t = (1-\delta)K_{t-1} + (Q_t - R_t). \tag{3.6}$$

The undepreciated portion of production capital is regarded as *installed* capital, which cannot be used for any purpose except production.

3.2 Equilibrium

The allocation of new investment between its alternative uses is performed by the business sector during each period. This is done in two sequential but timeless stages: the search stage, and the production stage. First, a production technology – indexed by θ – is found by either adopting a default technology that was used in the previous period, or by investing in search for a better technology. When further search seems unwarranted, the remaining capital is added to the installed capital and combined with labor to produce the output.

[14] The parameter A has a distinct role in the calibration, as is discussed below.

[15] This definition implicitly assumes that the trade deficit is entirely financed by unilateral transfers from abroad. While not exactly true, this simplifying assumption is not far from reality for the period under consideration in Israel.

As stated above, the entire business sector is aggregated into a single firm, which nevertheless behaves competitively in the labor market. The firm ends the search stage and enters the production stage with known levels of its production capital and technology, (K, θ), omitting the period subscript. The only decision left at that stage is the choice of labor input. Taking the wage rate w parametrically, the profit maximizing employment level for the firm is:

$$\ell^*(K, \theta, w) = K \cdot \left(A\theta \cdot \frac{1-\gamma}{w}\right)^{1/\gamma}. \tag{3.7}$$

The resulting profits are given by:

$$\pi(K, \theta, w) = \gamma \cdot K \cdot \theta^{1/\gamma} \cdot A^{1/\gamma} \left(\frac{1-\gamma}{w}\right)^{\frac{1-\gamma}{\gamma}}. \tag{3.8}$$

When search is conducted, the firm draws technologies from a time-invariant distribution, given by the Pareto distribution,

$$H(\theta) = 1 - \theta^{-\lambda}, \quad \theta \geq 1, \quad \lambda > 1. \tag{3.9}$$

Each successive draw from this distribution costs the firm α units which are paid out of the new resources made available to it at the beginning of the period.[16] The search strategy in the form of a stopping rule is chosen in order to maximize the expected current period profits in (3.8).[17] These profits are random as of the beginning of the period, being a function of the random results of the firm's search activity, (K, θ), and the wage rate.

Under the above assumptions, the search problem of the firm has the simple "reservation technology" character. The firm continues the drawing process until it finds a technology which exceeds a threshold function. This function depends on the technology which is available without search (the "fallback" technology), the firm's remaining search capital and on its already installed production capital. We denote the threshold technology by $\theta^*(q, (1-\delta)K, \theta_0)$, when q units of investment are still available for R&D and production, the installed production capital is $(1-\delta)K$ and the fallback technology is θ_0. We refer the reader to Appendix A, where it is shown that the threshold function which determines the sequential R&D investment process is the solution to the following recursive relation, for $q \geq \alpha$:

$$\theta^*(q, Z; \theta_0) = \tag{3.10}$$

$$\max\left\{\left[1 + \frac{1}{\lambda\gamma - 1}[\theta^*(q-\alpha, Z; \theta_0)]^{-\lambda}\right]^{\gamma} \theta^*(q-\alpha, Z; \theta_0)\left[1 - (\frac{\alpha}{q+Z})\right]^{\gamma}, \theta_0\right\}$$

where beginning of period installed capital is $Z \equiv (1-\delta)K$.

[16] The fact that the distribution of untried technologies is time-invariant implies, of course, that there is no exogenous improvement of the technologies. While it is relatively simple to posit some exogenous rate at which this distribution changes over time, we prefer to adhere to the pure endogenous nature of the growth process.

[17] As we have indicated, the single firm setting should be regarded as a simplified version of a multi-firm environment, in which the the impact of current R&D efforts on future opportunities is ignored.

Condition (3.10) results from the Bellman equation which equates the value of stopping the search with a technology level $\theta^*(q,(1-\delta)K,\theta_0)$, with the value of taking one more draw and then deciding whether to stop the search or keep on drawing. Obviously, the resulting threshold cannot fall below the technological default option, θ_0. Due to the Cobb Douglas production technology, the optimal search strategy is independent of the distribution of the wage rate.

The dynamic equilibrium can now be described as follows. At the beginning of period t, $q = Q_t$ from (3.5), and the installed capital $(1-\delta)K_t$ is given. After concluding its search according to (3.10) for that period, the firm knows its technology as well as the total amount of production capital given by (3.6). It then hires labor, taking the wage rate as given, according to (3.7). The equilibrium wage rate is determined by equating labor supply, (3.1), to its demand, (3.7). Output is produced according to (3.2). Given the exogenously determined net imports and government consumption, disposable income is computed by (3.3). The saving rate and the exogenous non-industrial investment determine the new capital made available to the business sector in the next period, (equations (3.4) and (3.5)).

Growth in this model is stochastic, and is driven by R&D which, on average, increases θ. However, sustaining the growth (in expected value terms) requires an ever increasing R&D investment. It can be shown that when $\gamma + 1/\lambda \geq 1$, growth can be sustained.[18]

3.3 Government subsidies

We assume that each period the government transfers to the business sector resources amounting to SUB_t. These subsidies are fully financed by taxes. Accordingly, the disposable income is amended to:

$$Y_t^D = Y_t + (M_t - X_t) - (G_t + SUB_t). \tag{3.11}$$

Government subsidies to the business sector affect the optimal search behavior. Both subsidy schemes we discuss provide additional incentives to R&D, but they

[18] According to our intertemporal spillover assumption, any technology discovered at period t becomes the "fallback" technology of period $t+1$. Therefore, a typical trajectory of our economy consists of phases in which search takes place, followed by phases during which the technology found in the previous phase is utilized without any attempt to improve upon it. During a no-search phase, the economy grows deterministically at decreasing rates. When $\gamma + 1/\lambda \geq 1$, this no-search phase ends within finitely many periods, and search efforts are renewed. While in a search phase, a limiting version of the search strategy given by (3.10) for large enough q can be used to derive a lower bound on the expected growth rate of output:

$$E\left\{\frac{Y_t^B}{Y_{t-1}^B}\right\} \geq \Delta \cdot (Y_{t-1}^B)^{\gamma+1/\lambda-1} \cdot \left(s + \frac{(1-\delta)K_{t-1}}{Y_{t-1}^B}\right)^{\gamma+1/\lambda}$$

where Δ is a positive constant which depends on γ, λ, α and A. The RHS of the inequality is clearly greater than $\Delta \cdot s$ when $\gamma + 1/\lambda \geq 1$. These observations generalize the results in Bental and Peled (1996) for the case of 100% depreciation, ($\delta = 1$).
The increased R&D investment is both feasible as the economy becomes richer, and more profitable as the results of R&D are applied to a larger capital base.

differ from one another in the precise way they affect the search strategy, and hence the resulting performance. We now elaborate on these differences in terms of the threshold function characterized by (3.10).

Unrestricted subsidy

With unrestricted subsidy, the amount of new capital available to the business sector at the beginning of the period is amended from (3.5) to:

$$Q_t = S_t - IX_t + SUB_t. \tag{3.12}$$

Thus this subsidy increases the beginning of period resources that can be allocated at will to R&D or production. From (3.10), this will raise the threshold function used by the searching firm, (holding Z fixed). Consequently, we expect this policy to result in more R&D investment, and higher output growth than would be the case absent subsidies. The increased growth stems from higher technology levels and possibly higher levels of production capital at the conclusion of the search phase.

Search capital subsidy

This policy effectively forces the firm to first spend the subsidy amount on search at the beginning of the period, thus possibly improving upon the default technology θ_0 in (3.10). Specifically, the firm will conduct $[SUB_t/\alpha]$ draws up front, and set θ_0 to the best technology among those sampled and the original default technology for that period. It is likely, therefore, that θ_0 in this case will be larger than it is under the previous subsidy. However, after completing this government funded phase of its search, the firm has less resources compared to the amount (including the subsidy) available to it under the previous scheme. Consequently, the search policy chosen at this stage is different across these two regimes.

The difference between the two schemes is best illustrated by considering the following (frequent) realization of the economy's growth path. Suppose that a substantially improved technology has been discovered during the search phase at a particular period. From its point of view, the firm may find search unwarranted for several subsequent periods, given the level of the recently discovered technology and the resulting output growth. However, if search activities resulting in further technological improvements continued, growth during the same period might have been higher due to the intertemporal spillover effects (which are ignored by the firm). Such continued R&D efforts will be induced by R&D-specific subsidies. On the other hand, to the extent that the original technological discovery is indeed very significant, this subsidy-induced search effort may fail to result in further technological improvements. Consequently, resources may end up having been "wasted", and the growth rate may be negatively affected. Accordingly, the actual relative growth effects of the two subsidy schemes depend on the economy's specific parameter values, and needs to be evaluated numerically.[19]

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[19] Another subsidy that is commonly implemented earmarks the transfers to production capital formation. In this case the searching firm will typically end up substituting the subsidy for the resources it

4 Calibration

Our goal is to compare different restrictions associated with given amounts of subsidies to the business sector, holding everything else constant. The subsidy levels used in our computations are 30% of the R&D expenditures generated by the model absent any subsidy. The 30% rate corresponds to the rates stipulated by the "law for promotion of capital investment" in Israel, under which the government operated during the sample period, and which covered a broad range of industrial activities, including R&D. The precise choice of the remaining parameter values and of the initial conditions used in our calibration is explained later in this section.

4.1 Basic national accounting

The basic time series we use as a guideline for the purpose of calibrating the model is that of the output of the business sector in Israel, 1975 – 1990. The aggregate real disturbances during this period are relatively small.[20] Business output grew over this period at a (geometric) annual average rate of 3.85%. Output was 21.5 billion shekels, (NIS), in 1975, and 37.9 billion NIS in 1990 (all in 1986 prices, see Appendix B).[21] These values are assumed to have been generated by a government policy approximated by our *unrestricted* capital subsidy.

Disposable income is divided into the following components: the output of the business sector, the output of the public sector, net imports, and taxes needed to finance all government activities:

$$Y^D = Y^B + Y^G + (M - X) - (G + SUB), \tag{4.1}$$

where Y^B is the output of the business sector and Y^G is the output of the public sector.[22]

Of the ingredients of disposable income, Y^B, the output of the business sector is endogenous to our model, whereas SUB is determined by us as modelers. The remaining elements are treated by us as fixed in the sense that they do not directly respond to subsidies to the business sector. The series of $Y^G + (M - X) - G$

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would have anyhow allocated to production capital formation. Therefore, for all practical purposes, the result will be equivalent to that of the unrestricted subsidy described above.

[20] The period was characterized by major nominal disturbances, with inflation peaking at 30% per month in June of 1985 and a stabilization program which followed. However, the subsequent years beyond 1990 were characterized by several very important real shocks. A major immigration wave has started in 1991 and increased Israel's population by almost 20%. In addition, the political developments in the Middle East at that time were thought by many to be the beginning of a new historic era. Finally, the 1990's witnessed the major outburst of the so-called high-tech based "New Economy".

[21] The average exchange rate in 1986 was 1.5 new Israeli shekels, (NIS), per $US. From this point we measure all relevant magnitudes in billions of 1986 shekels.

[22] A significant portion of economic activity was generated in Israel during the period by the government, which directly owned public utilities, some industrial conglomerates, the railroad company and the national airline, as well as several industrial and commercial banks. Although the public sector was responsible for up to 45% of GDP at the beginning of the sample period, (with that share declining monotonically), we choose to focus on the output of the business sector, which seems to be more responsive to profit maximization than the government owned sector.

during the sample period does not display any clear pattern (see Appendix B), so we set its value in the calibration to its sample average of 5.069.

In order to find the amount of new resources made available annually to the business sector, Q in (3.12), we use the commonly estimated private sector saving rate of 0.3 out of disposable income to compute private savings, and subtract the non-business investment, IX_t (mainly government investment in infrastructure). We treat the latter form of investment as exogenous. Again, in the absence of a clear pattern, we use the sample average of 3.83 for IX_t for each year in our calibrations.

4.2 Parameters choice and calibration method

As stated above, the production function is given by a constant returns to scale Cobb-Douglas function,

$$F(K, \ell; \theta) = \theta A K^{\gamma} \ell^{(1-\gamma)}, \tag{4.2}$$

where K denotes the production capital, and ℓ denotes labor input chosen by the firm.

For some parameters we use available estimates and commonly used values. In particular, the depreciation rate of capital, δ, is set to 6% annually, and the growth rate of the labor force, x_L, is set to its average over the 1975–90 period of 1.7% per year. Remaining to be specified are the parameters: A, α, λ, γ, the initial values for the first period (1975): $(1-\delta)K_0$, Q_0, ℓ_0, and θ_0, and the subsidies to the business sector in each of the following periods. We choose these values to satisfy three requirements, under the assumption that the data observations were generated by a regime close to our "unrestricted subsidy":

(i) The levels of resources generated by the model in the first period, 1975, should match their counterparts in the data. Specifically, we choose parameters and initial conditions so that the model generates employment, output and installed capital levels in the business sector, similar to the data observations for those variables in 1975, (0.9 million workers, 21.5, and 45.2 billion NIS, respectively).[23]

(ii) The parameters should be theoretically capable of generating sustained growth in output per worker in the business sector.

(iii) The expected output level generated by the model for 1990 should be as close as possible to the output level of the business sector in the data for that year, 37.9.

We now proceed to describe the precise way these parameters and initial values were chosen. The parameter values of γ and λ are jointly determined by the minimal restriction necessary to sustain growth:

$$\gamma + 1/\lambda = 1. \tag{4.3}$$

We set $\ell_0 = 0.9$. For any given λ, we set γ according to (4.3). We choose K_0, and Q_0 so that if there is no investment in R&D at the initial calibration period

[23] See data in Appendix B

and all the “new” capital is used for production, the level of production capital in that period, $(1-\delta)K_0 + Q_0$ approximates the observed level of 45.2. The idea behind initializing the calibration by making the business sector close to indifferent between searching and not searching in that period, is that we want the search effort (measured by its costs relative to output) in that period to be close to the search efforts generated by the model in subsequent periods. For the business sector to be indifferent between searching and not searching in the initial period, we set θ_0 to be the threshold technology for the aforementioned level of capital. We then set the parameter A so that using technology θ_0 with the available resources, output levels in that period approximate the observed value of 21.5.

The above procedure requires that the sampling cost, α, be specified. Lacking any data on this cost, we try three different values for that parameter which span a reasonable range for its possible values, (more on this below). Using this procedure we select, for each value of α, the value of λ which approximates well the desired expected output at 1990 dictated by the data for that year. With these parameters and initial values set, we run the model 40,000 times over the entire calibration period, (16 years), under the “no intervention” regime, and set the subsidy levels for each of the calibration periods at 30% of the average amounts spent on search in that period.[24]

The three values of α we tried are: 0.1, 0.5, and 0.9. These values correspond to a sampling cost of 100 to 900 million NIS in 1986 prices, which constitute 0.26% to 2.37% of the annual output of the business sector in 1990. These seemingly high values of α need a clarification. One should remember that we consider a single “aggregate” searcher, whereas there were about 200 firms officially registered as being involved in R&D in Israel in 1990. Their search efforts were conducted independently, while our single searcher aggregates all their search activities. Accordingly, when dividing the range of 100 to 900 million NIS for α by 200 firms, we obtain a search cost per firm of 0.5 to 4.5 million NIS. We believe that this range is broad enough to include any reasonable prior on this parameter.[25]

For each of these three values for α, we follow the above procedure for selecting values for the other parameters. The resulting parameter values are listed in Table 1 below. In particular, we find that for α=0.1, we need $\lambda = 2.4$, which implies via (4.3) $\gamma = 0.583$. The corresponding values for λ and γ when $\alpha = 0.9$ are 2.3 and 0.565, respectively, while those for the intermediate value of the sampling cost lie in between.[26]

For each set of parameters, we run the model 40,000 times, starting with the same initial conditions. This number was chosen to be high enough to allow us to

[24] Clearly, this procedure which pre-sets the subsidy levels by using R&D expenditure absent government intervention, underestimates the subsidy under active government policies. Our calibration confirms that this difference is negligible.

[25] To get a feel for this range for α, it can be translated to R&D worker-years. Using the average employer’s cost of 100,000 NIS per R&D worker, the above range for α corresponds to 5 to 45 R&D worker-years per draw for a single firm. This range covers the scope of most industrial R&D projects.

[26] Notice that the value of γ exceeds the capital share in income of about 0.3. Similar deviations have been observed and commented upon, among others, by Romer (1987), Barro and Sala-i-Martin (1992), and Mankiw, Romer and Weil (1992).

gauge the statistical significance of differences in sample moments across different policies. The implied high number of runs reflects the high variance of sample paths generated by the model in light of the relatively low value of λ, which is dictated by the calibration.

To summarize, we use $\beta = 0.3, \delta = 0.06$, subsidy rate $= 30\%$, and $(\alpha, \lambda, \gamma) \in \{(0.1, 2.3, 0.565), (0.5, 2.32, 0.569), (0.9, 2.4, 0.583)\}$

5 Comparing alternative policies

5.1 Method and results

Table 1 reports averages of five variables over a sample of 40,000 runs for the entire sample period under three alternative policies: "no intervention", "unrestricted subsidy", and "R&D subsidy". The five averages reported in the table are:

(i) Mean level of output of the business sector in 1990 (this was the calibration target which determined the parameter choices).
(ii) Mean level of the capital stock in the business sector in 1990.
(iii) The geometric annual growth rate of total output in the business sector over the period 1975–1990.
(iv) Mean annual growth in total factor productivity (see below).
(v) Mean annual share of search costs out of the output of the business sector over the period 1975–1990

Table 1. 16-years data and calibration means (standard deviations in brackets, N=40,000)

Variable	Y^B_{1990} [a]	Z^B_{1990} [b]	g_y [c]	TFP [d]	$\frac{\text{R\&D}}{Y^B}$ [e]
Data	37.93	71.56	3.85	1.28	5.30
Parameters: $\alpha = 0.1, \lambda = 2.40, \gamma = 0.583, A = 0.1238$					
Intervention free	35.73 (0.64)	61.15 (0.70)	3.52	0.83 (0.02)	6.09 (0.58)
Unrestricted subsidy	38.31 (0.67)	65.15 (0.78)	3.98	0.98 (0.02)	7.24 (0.76)
R&D subsidy	39.11 (0.79)	65.61 (0.91)	4.12	1.00 (0.02)	7.27 (0.72)
Parameters: $\alpha = 0.5, \lambda = 2.32, \gamma = 0.569$ $A = 0.2156$					
Intervention free	35.23 (1.05)	60.48 (1.42)	3.45	0.73 (0.02)	6.25 (0.55)
Unrestricted subsidy	39.66 (1.22)	65.82 (1.46)	4.25	0.89 (0.02)	7.44 (0.67)
R&D subsidy	36.63 (0.71)	62.76 (0.89)	3.70	0.86 (0.02)	7.47 (0.65)
Parameters: $\alpha = 0.9, \lambda = 2.30, \gamma = 0.565$ $A = 0.2696$					
Intervention free	35.39 (1.29)	59.68 (1.05)	3.47	0.70 (0.02)	6.33 (0.50)
Unrestricted subsidy	38.86 (1.20)	65.03 (1.47)	4.14	0.88 (0.02)	7.60 (0.70)
R&D subsidy	43.09 (3.10)	67.65 (3.06)	4.86	0.91 (0.03)	7.64 (0.65)

[a] Output of the business sector, (billions NIS in 1986 prices).
[b] Capital stock of the business sector, (billions NIS in 1986 prices).
[c] Output growth, (geometric annual rate, 1975–1990, %).
[d] Total factor productivity growth, (annual average, 1975–1990, %).
[e] R&D expenditure share of output, (annual average, 1975–1990, %)

For each of those variables, we first report its data analogue, and then its sample average in our runs, under each of three alternative parameter configurations. Standard errors of the simulation mean statistics, $\frac{\sigma_{\bar{x}}}{\sqrt{N}}$, are reported in brackets.[27]

We have measured the annual total factor productivity growth, (TFP), as follows. For each of the 40,000 runs we compute the improved productivity in each of the 16 calibration periods in the usual way. With $Y_t = A\theta_t K_t^{\gamma} L_t^{1-\gamma}$, we have $\Delta Y/Y \cong \Delta\theta/\theta + \gamma\Delta K/K + (1-\gamma)\Delta L/L$, so that the measure of improvement in technology over time is approximated by:

$$\Delta\theta/\theta \cong (Y_{t+1} - Y_t)/Y_t - \gamma(K_{t+1} - K_t)/K_t - (1-\gamma)(L_{t+1} - L_t)/L_t.$$

We average this measure over the calibration period, and report the sample mean of this statistic across all 40,000 runs.

The statistical significance of differences in the reported sample means is provided in Table 2. Here we compare the significance of the economy's performance under the alternative policies, and the accuracy of our calibration against the data. Specifically, we use the standard deviations of the sample means to determine whether sample differences between the output level in 1990, the stock of capital in 1990, and the annual TFP growth are statistically significant.

Table 2. Significance across policies: t-statistic values Sample size $= 40,000$

Policies	Variable	α =0.1 $\lambda = 2.40$ $A = 0.1238$	$\alpha = 0.5$ $\lambda = 2.32$ $A = 0.2156$	$\alpha = 0.9$ $\lambda = 2.30$ $A = 0.2696$
Unrestricted Sub.	TFP Growth	5.61[a]	5.38[a]	6.11[a]
vs.	Y^B_{1990}	2.79[a]	2.75[a]	1.98[b]
Intervention Free	Z^B_{1990}	3.82[a]	2.62[a]	2.97[a]
Unrestricted Sub.	TFP Growth	0.71	1.07	0.85
vs.	Y^B_{1990}	0.77	2.15[b]	1.27
R&D Sub.	Z^B_{1990}	0.38	1.78[c]	1.20
Unrestricted Sub.	Y^B_{1990}	0.61	1.44	0.80
vs. Actual Data	Z^B_{1990}	8.31[a]	3.95[a]	4.48[a]

[a] Significance level of 0.995 or higher.
[b] Significance level of 0.975 or higher, but less than 0.995.
[c] Significance level of 0.95 or higher, but less than 0.975

5.2 Discussion

The main finding in Table 1 is that the subsidies, regardless of the associated restriction, increase annual growth rates over the simulated period by 0.5 percentage points or more, relative to the intervention free regime. Figure 1 presents the model's predicted mean output level over the simulation period for $\alpha = 0.5$, along with its

[27] Since the growth rate reported is computed using sample means, we do not have a good estimate for the variance of this variable.

one standard error band, against the actual observations (recall that the calibration assumed that the data was generated under the unrestricted subsidy regime). It shows quite clearly the dominance of the two policy-affected paths relative to the intervention-free one. It also shows that the model captures reasonably well the trend of the growth path, albeit not its fluctuations.[28]

The growth impact of the subsidies is achieved with a moderate tax on income, amounting to less than 1.7%.[29] One way to appreciate the trade-off involved in higher growth rates obtained through higher taxes is to consider the length of time it takes to recoup the *after-tax* income level when a tax-financed subsidy of 1.7% on income per year permanently increases the growth rate of income by 0.5%. This period amounts to less than 5 years.[30]

The second conclusion from Table 1 is that both R&D-restricted and unrestricted transfer policies increase the annual TFP growth by about 0.15%, relative to the intervention free regime. This difference is highly significant. Thus, slightly less than a third of the total 0.5% gain in the growth rate of output during the calibration period can be directly attributed to technological improvements. The other two thirds of that gain are due to the fact that production capital accumulates at a faster rate. However, because of the dynamic complementarity between capital accumulation and technological improvement entailed in our model, the above TFP measurement underestimates the contribution of technology to growth, as explained in the next subsection.

The third conclusion from Tables 1 and 2 is that the differences in the TFP growth between the two active policies are highly *insignificant*. As argued above, this finding cannot be expected on *a priori* grounds, as the "R&D restricted" subsidy regime forces the business sector to spend the transfers on additional draws, even in periods when it may have preferred to use those resources for production.

The fourth, and rather surprising result to emerge from our calibration, is that all the above conclusions are robust to the magnitude of the sampling cost α. Despite the broad range of values we use for this parameter, 0.1 to 0.9, neither the values of other parameters (e.g. λ and γ), nor the performance measures of the economy,(e.g. the R&D share of output, or the effect of different policies), seem to be very sensitive to its magnitude, as long as the calibration goals are respected.[31]

The final conclusion we draw from Table 1 is that the model performs reasonably well even with respect to variables that were not used in the calibration process. Recall that only the mean output in 1990 under the "unrestricted subsidy" regime

[28] The one standard error band for output when $\alpha = 0.1$ is much narrower, due to the higher λ implied by the calibration. The opposite happens when $\alpha = 0.9$, where the R&D-subsidy induced growth rate is higher than the one induced by unrestricted subsidy, along with larger standard errors.

[29] The subsidy level in the model is in the order of 0.5 billion NIS, out of an average output level of about 30, (about 1.7%). This figure is lower than 30% of the average share of search costs out of output of about 7.5%, which amounts to 0.675 billion NIS. This difference is due to the fact that the mean of $\frac{\text{R\&D } COST}{OUTPUT}$, (which is about 0.075 in our simulations), is higher than the ratio of the means, when both are random.

[30] The payback period is calculated by $0.983 \cdot (1.0385)^t = (1.0345)^t$, which implies $t \cong 4.4$.

[31] In the following section we demonstrate that with different parameter values, which do not necessarily produce sample moments that match observations, the model is capable of producing rather different conclusions from those listed here.

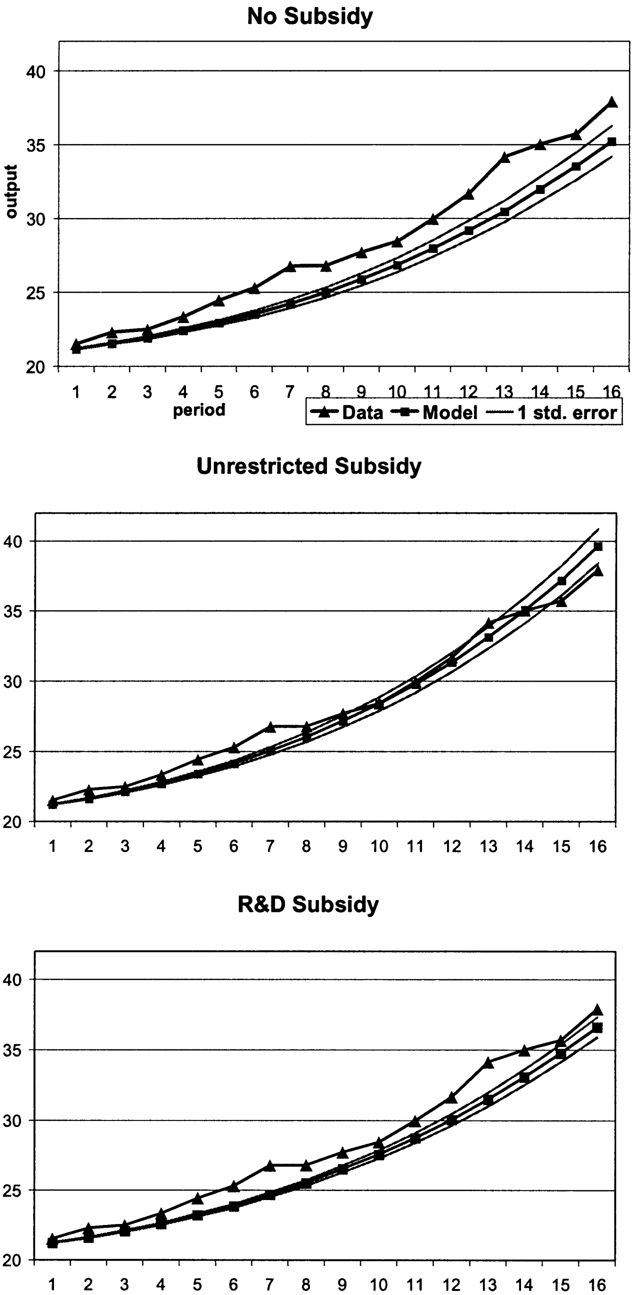

Fig. 1. Actual and model mean output(α = 0:5)

was used to select a value for λ. Yet, the other moments reported in Table 1 came out surprisingly close to their counterparts in the data. In particular, the model TFP growth rates compare well with the data. For the three values of α considered we get an average annual TFP growth during the period 1975–1990 of about 0.9–1.0%. The Bank of Israel estimates the TFP growth for that period at about 1.11%.[32]

Likewise, the sample means of R&D share of output in our runs under the unrestricted policy are between 7.2% and 7.6%. These figures can be compared to the reported average R&D cost share in the Israeli industrial sector, which, during the second half of the 1980s, was 5.3% on average.[33] The dynamic behavior of the R&D share in the model and in the data is similar: in both cases the share is increasing over time (in the data it is 2.5% in the early 1980s). However, the rate of increase predicted by the model seems to be too high. This, together with the somewhat lower than actual TFP growth rate reported above, indicates that we underestimate the productivity of R&D. These related results may be due to the fact that our model assumes that all technological improvements are the results of domestic investment in R&D, and ignores any international spillovers of such improvements, which are likely to be quite important.

Another consequence of the under-estimation of the productivity of R&D as measured by TFP growth is the lower than actual level of installed capital generated by the model (see column Z^B_{1990} in Table 1). Instead of accumulating physical capital, too much of the resources are diverted to R&D. However the model generates quite accurate capital output ratios for the business sector, both on average and dynamically. The actual average capital output ratio in the Israeli business sector, (excluding construction), declined from an average of 2.33 for 1981–1985, through 2.11 for 1986–1989, to 1.89 in 1995. In our simulations the corresponding ratio declines over time, from 2.2 to 1.9 in the same period.[34]

5.3 Decomposing the growth effects of subsidies

As noted above, our model implies a kind of dynamic complementarity between capital accumulation and technological improvement. Here we suggest a method of decomposing output growth into its capital and technology components which reflects this complementarity.

Naively, the higher rate at which capital grows may be attributed to the forced saving feature of the subsidy and to the increased voluntary saving generated by the increased output. However, absent technological change, a *fixed level* subsidy has only temporary growth effects, and capital and output growth rates converge back to

[32] Table B-15, (Appendix), The Annual Report of the Bank of Israel, 1996.The Bank of Israel figure is also an average, with a substantial standard deviation of 2.37%.

[33] The Israeli Central Bureau of Statistics reports with some gaps the GDP of the industrial sector and the R&D expenditure of firms employing 50 workers or more in that sector. The average of 5.3% is based on data reported for 1985/6, 1987, 1988, 1990, and 1991, and the ratio between total R&D expenditure in the industrial sector, (reported only recently), and that of firms employing 50 workers or more, of about 1.1.

[34] Table B-3, The Annual Report of the Bank of Israel, 1995. The difference between the average capital-output ratio and the ratio of the average capital to average output explains why the model fails on the latter measure while doing well of the former, (see footnote 29).

the growth rate of the labor force. In contrast, when the subsidy generates persistent technological changes as it does in our model, those subsidy-induced effects on the growth rate of output and production capital do not fade away. Accordingly, the effective growth contribution of technological improvements is greater than that measured by TFP growth.

The fact that standard measures of TFP growth underestimate the growth contribution of subsidy-induced technological improvements can be demonstrated even within our 16 periods calibration. The parameters we have used imply that the model economy starts below the steady-state associated with the initial technology. Therefore, a subsidy to capital formation has a short-run growth effect even when the technology remains fixed. The difference between the measured growth impact of the subsidy in a model with endogenous technical changes, and the growth effect the same subsidy induces when the technology is being held fixed at its initial level, can be interpreted as the effective short-run contribution of the subsidy through technological improvements. This difference can be calculated as follows.

A deterministic version of our model, absent technological improvements and R&D investment, can be written as:

$$(1 + x_L)k_{t+1} = (1 - \delta)k_t + \beta(y_t + yx_t - sub_t) + (sub_t - ix_t) \tag{5.1}$$

where $k = \frac{K}{L}$, $y = \frac{Y^B}{L} = A\theta_0 k^\gamma$, $yx = \frac{Y^G+(M-X)-G}{L}$, and all other lower case variables are per capita analogues of the level variables used before, (see sections 3 and 4) . We use the same initial conditions and parameter values that were used in our calibration, and compare the evolution of this deterministic economy under two policies: a fixed level tax-financed subsidy of 0.5 units of output per period, implying: $sub_t = 0.5/L_t$, and an intervention free policy, implying $sub_t = 0$.[35]

For the intermediate value of the sampling cost, ($\alpha = 0.5$), total output level increases from 21.21 in period $t = 1$ to 31.16 in period $t = 16$ *with* subsidies, while the corresponding output levels *without* subsidies are 21.12 and 29.81.[36] These output levels represent annual geometric growth rates of 2.60%, and 2.32%, with and without subsidy, respectively. Consequently, the subsidy generates an annual growth differential of 0.28% during the first 16 periods in the fixed technology model, compared to 0.5% found in our model with endogenous technological change. Therefore, the growth effect of the subsidy which should be attributed to its impact on technological improvements amounts to $0.5 - 0.28 = 0.22\%$, compared to the TFP growth of about 0.15%, reported above.[37]

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[35] We use a fixed level of subsidy per period of 0.5, which is close to the average subsidy used in our calibrations, (see f.n. 29).

[36] Note that the resulting output levels for 1990, (t=16), are based on the assumption that technology remains fixed at its 1975 level, and are consequently lower than those reported in Table 1. The results of this thought experiment, obtained with the parameter values used in conjunction with the other two values of α, are very similar, and are not reported here.

[37] A measure of how far our economy is from a fixed technology steady state, given our parameters and initial conditions, is provided by the output *per worker* The value of this variable in (5.1) for $t = 1$ is 23.0, and for $t = 16$ it is 26.9 with subsidies, and 25.7 without subsidies. The annual growth in output per worker, which is transient given a fixed technology, (and amounts to an annual average of about 1% in the first 16 periods), fades away as output per worker converges to its steady state value of about

6 A robustness test

The main result we have found above is that under parameter values dictated by the calibration, the differences between the impact of an R&D-earmarked and an unrestricted capital subsidy are statistically insignificant. To test whether this implication is specific to the calibrated economy rather than an artifact of the model, we examine these two policies under different parameter values. Clearly, with the new parameters our model no longer matches the key features of the Israeli business sector.

We focus on the growth performance of our model economy under alternative values of the key parameter characterizing the distribution of untried technologies, λ. This parameter determines how productive the search for technological improvements is, and consequently it has a crucial influence on the R&D behavior of firms in our model. Through the assumed "total constant returns to scale", (see (4.3)), the capital share of output, γ, is also determined by λ . In order to identify the effect of changes in λ (and γ), we keep the subsidy *amounts* fixed as we vary that parameter. In particular, we use here the *same* subsidy amounts that are used in the corresponding calibrated versions of the model. Since the R&D-earmarked subsidy forces firms to spend resources on search, even when there is hardly any chance to improve upon the exiting technology, we expect the two subsidy policies to produce different results under high values of λ , when the mean of the distribution is low and its tail diminishes relatively quickly. And conversely, when λ is very small, implying a highly productive R&D, we expect even smaller differential impacts of the policies considered, because firms have enough incentives to invest "free" resources in R&D.

Table 3 below reports the growth results for various combinations of λ and the sampling cost α. In particular, we compute the effects of the policy for $\lambda = 2.02$, (the variance of the distribution is finite only when $\lambda > 2$), and for $\lambda = 30$, (hardly any chance to find an improvement), and compare those to the corresponding results of the calibrated economy, reported in Table 1 above.

Table 3. Annual Growth Rates under Different Parameters (%) Sample size = 40,000

α	λ	Average annual growth of output (%)		
		Intervention free	Unrestricted sub	R&D-earmarked sub
0.1	2.02	1.16[a]	1.08[a]	1.11[a]
0.1*	2.4*	3.52	3.98[a]	4.12[a]
0.1	30.0	4.84	5.47	5.19
0.9	2.02	1.05[a]	1.18[a]	1.74[a]
0.9*	2.3*	3.47	4.14[a]	4.86[a]
0.9	30.0	4.84	5.48	5.11

* Results of a calibration parameter configuration, also reported in Table 1.
[a] Differences among so-marked growth rates in same row are statistically insignificant at 95% or above

55.15. This steady state limit of output per worker is common to both policies because the subsidy *level* is fixed.

When λ is high the model behaves like an "Ak" model: the value of γ implied by (4.3) is close to 1, and R&D activities are unlikely to find a superior technology. Accordingly, the R&D-restricted subsidy induces *wasteful* R&D investments. Moreover, the tax involved in raising the funds for the subsidy reduces the effective saving rate, which, in an "Ak" model, reduces the growth rate. In contrast, an unrestricted subsidy in this case amounts to a capital-subsidy which, in effect, increases the saving rate and therefore also the growth rate. These effects are particularly evident when the search is unproductive and its cost is high, ($\lambda = 30,\ \alpha = 0.9$). The other extreme case of highly productive and relatively inexpensive search, ($\lambda = 2.02,\ \alpha = 0.1$), results in very similar growth impact of the two subsidies: firms do not need the restriction to allocate the subsidy to search. The significance of the differences among the two subsidies in the mixed cases is determined by the variance of the distribution of technologies: when its variance is large, ($\lambda = 2.02$), differences are insignificant, and when it is small, ($\lambda = 30$), those differences are significant, and result in a better growth performance under the unrestricted subsidy. We conclude, therefore, that the restriction on how firms may use the subsidy could matter. However, when it does - the restriction hurts economic growth.[38]

The two subsidy schemes produce similar TFP growth results, with statistically insignificant differences in most cases. The R&D-earmarked subsidy produces a slight but statistically significant higher TFP growth only in the limiting case of almost no uncertainty in R&D, ($\lambda = 30$).[39] Interestingly, however, TFP results are not monotone in the productivity of search represented by λ. For both small and large values of λ all the policies, (including the intervention-free) produce only a tiny growth in TFP, an order of magnitude smaller than those reported in Table 1 for the calibrated parameters.[40]

These calculations demonstrate that our original "equivalence" result of the two policies considered does not hold in general. However, the numerical illustrations also explain the difficulty in providing an intuitive explanation for the fact that the externality embedded in the model does not necessarily imply that subsidies should be directed towards R&D. Our results seem to imply that when search is productive and when firms are given sufficient resources, the amount of resources the firms choose to allocate to search activities is not much different from the amount we give them through the R&D-specific subsidy. Corresponding to that, in such cases the restriction makes little difference. But when search is less productive – such restrictions on the use of the subsidy are counter productive.

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[38] The fact that more productive R&D, (lower λ), results in lower growth, holding α fixed, is a consequence of the maintained linkage between λ and the capital elasticity of output, γ, (see equation (4.3)).

[39] This is an implication of the fact that R&D in this case is "forced" on the firm, in the sense that R&D productivity is so low that the firm would choose not to spend the subsidy for R&D if it were free to do so.

[40] The result is quite intuitive for high values of λ. It seems that when λ becomes small, the inducement to conduct R&D is just "too big", and in this sense, tere is "too much" search. Notice also the low growth rate in Table 3 when $\lambda = 2.02$.

7 Conclusion

This paper demonstrates how an endogenous growth model with explicit micro underpinnings can be used to obtain meaningful quantitative growth implications of alternative subsidy policies. For the Israeli economy, we have identified most parameters by either directly observing their value, (saving rate, subsidy rate and initial conditions), or by matching simulated moments to their observed counterparts. Lacking any direct way to assess the value of one of the crucial parameters in our model, the sampling cost α, we proceeded by calibrating the model for a broad range of values for it. The remaining two parameters on which there is no direct evidence are the scale parameter of the technology, A, and the single parameter of the Pareto distribution over which search for new technologies is conducted, λ. The values of these parameters are tightly determined by the requirement that the model's predictions for the final period should closely match their observed counterparts under the assumption that the latter were generated under the "unrestricted subsidy" regime. Surprisingly, the calibrated values for these parameters turned out to be almost insensitive to the sampling cost, despite a factor of 9 assumed between the highest and the lowest α. Since the calibration is based on one of the three regimes considered, it is remarkable that the growth rates under the other two policies are also insensitive to the choice the search cost parameter.

Our analysis clearly indicates that in the economy we studied, growth promoting subsidies have a quantitatively significant long-run impact. Moreover, this impact is evident and similar for both the unrestricted subsidy and R&D subsidy. It manifests itself in statistically significant higher output levels and annual TFP growth rates.

The lasting growth effects of policies fostering capital accumulation depend crucially on the endogenous process of technological improvements assumed by us. Absent such a process, these subsidies would only have transient and declining growth effects. Subject to our model's mechanism of endogenous growth, the general subsidy has lasting growth effects because the private sector finds it optimal to allocate some of its additional resources to finance higher levels of R&D investment. As a matter of fact, the similarity between the impact of the two policy measures indicates that (at least in Israel) there is no need to tie the subsidies to R&D activities, as the private sector finds it in its interest to allocate additional resources to R&D.

The model seems to overestimate the R&D investment needed to generate the observed growth rates of the Israeli business sector. This is a likely result of the fact that the only source of growth in the model is self-generated R&D. It may be interesting to extend the analysis and allow foreign spillovers to impact the domestic economy.

Finally, one should note that our results are positive and not normative. We used the criterion usually employed in practice to compare policies, namely the rate of the economy's growth. However, one may ask normative questions concerning the optimality of various policy measures, or of the optimality of any particular subsidy (and corresponding tax) rate. To do so a model with explicit dynamic welfare considerations will have to be developed.

Appendix

A. Optimal search strategy

Let $\varphi(q, K, \theta)$ be the profit to the searcher's when the technology θ is operated with the installed capital, $(1-\delta)K$, plus the remaining q units of new capital. Given the assumed Cobb Douglas production function, $A\theta k^{\gamma}\ell^{1-\gamma},\ 0<\gamma<1$, when labor is hired optimally at the wage rate w, we have:

$$\varphi(q, K, \theta) = A^{1/\gamma}\gamma \cdot \left(\frac{1-\gamma}{w}\right)^{(1-\gamma)/\gamma} \cdot (q + (1-\delta)K)\,\theta^{1/\gamma}. \tag{A.1}$$

The searcher seeks to maximize the expected value of $\varphi(\cdot)$ by choosing a strategy that maps sampled technologies and remaining new capital into the binary decision "accept" or "reject". At each stage during the search process the searcher can choose a default technology option, θ_0. Accepting a technology means stopping the search and operating that technology with all available capital, $q + (1-\delta)K$. Rejecting a sampled technology means making at least one more draw, at the cost of α units of new capital. The search is conducted over draws from the distribution $H(\theta), \theta \in [\underline{\theta}, \overline{\theta}]$. As noted in the text, we choose the Pareto distribution, $H(\theta) = 1 - \theta^{-\lambda}, \quad \theta \in [1, \infty], \quad \lambda \geq 1$.

The Bellman equation that summarizes the optimal decision is:

$$V(q, K, \theta; \theta_0) = Max\left\{\varphi(q, K, \theta)\varphi(q, K, \theta_0), EV(q-\alpha, K, \tilde{\theta}; \theta_0)\right\}, \tag{A.2}$$

where the expectations are taken with respect to the random result of the new draw, $\tilde{\theta}$. Solving (A.2) yields the optimal search strategy, to be denoted $\theta^*(q, K; \theta_0)$, such that the search process is stopped, and the technology θ is utilized with $q+(1-\delta)K$ units of capital, as soon as $\theta \geq \theta^*(q, K; \theta_0)$.

Since $\varphi(q, K, \theta)$ increases in θ, (see (A.1equating the two terms in the maximand in (A.2), utilizing the fact that when $\theta \geq \theta_0$:

$$V(q-\alpha, K, \theta; \theta_0) = \begin{cases} \varphi(q-\alpha, K, \theta), & \text{if } \theta \geq \theta^*(q-\alpha, K; \theta_0) \\ \varphi(q-\alpha, K, \theta^*(q-\alpha, K; \theta_0)), & \text{if } \theta < \theta^*(q-\alpha, K; \theta_0). \end{cases} \tag{A.3}$$

In particular, (A.3) implies:

$$\begin{aligned} EV(q-\alpha, K, \tilde{\theta}; \theta_0) = {} & H\left[\theta^*(q-\alpha, K; \theta_0)\right] \cdot \varphi\left(q-\alpha, K, \theta^*(q-\alpha, K; \theta_0)\right) \\ & + \int_{\theta^*(q-\alpha, K; \theta_0)}^{\infty} \varphi(q-\alpha, K, \theta)\, dH(\theta) \end{aligned} \tag{A.4}$$

Equating the two terms in the maximand of (A.1), using (A.4), together with the particular specification of $\varphi(\cdot)$ and $H(\cdot)$, we get $\theta^*(q, K; \theta_0)$ as the solution to the recursive relation:

$$\theta^*(q,K;\theta_0) = \left(1 - \frac{\alpha}{q+(1-\delta)K}\right)^{\gamma} \cdot \tag{A.5}$$

$$\left\{[1-\theta^*(q-\alpha,K;\theta_0)^{-\lambda}]\theta^*(q-\alpha,K;\theta_0)^{1/\gamma} + \lambda \int_{\theta^*(q-\alpha,K;\theta_0)}^{\infty} \theta^{1/\gamma-\lambda-1} d\theta\right\}^{\gamma}$$

Equation (A.5) allows us to recursively solve for $\theta^*(q,K;\theta_0)$. Specifically, given any initial quantity of new capital, q, at most $n(q) = [\frac{q}{\alpha}]$ draws from H can be taken. We let $q_0 \equiv q - \alpha n(q)$, $\theta^*(q_0,K;\theta_0) = \theta_0$, and use (A.5) to find $\theta^*(q_0 + n\alpha, K;\theta_0)$, $n = 1, 2, ..., n(q)$. This procedure yields equation (3.10) in the text, where the remaining new capital is simply denoted by q.

B. Israeli data

Israeli data, 1975–1990, in 1986 millions of new shekels, (NIS), rounded. N^B is in thousands of workers, rounded.

Year	Y^B	Y^G	G	M–X	I^G	I^D	YX	IX	K^B	N^B
1975	21533	10448	13299	8924	1446	3278	6072	4724	45173	877
1976	22308	10185	12086	7120	1191	2993	5219	4184	48500	882
1977	22509	10601	10507	5455	1070	2358	5549	3428	47271	903
1978	23364	11070	11651	6702	1160	2304	6120	3464	49162	934
1979	24462	11522	10621	6867	504	2569	7767	3073	51472	957
1980	25318	11782	14590	4947	889	3386	2139	4276	54097	962
1981	26786	11984	15538	6064	912	3457	2510	4369	55828	977
1982	26813	12383	14575	7303	947	3347	5111	4294	57447	992
1983	27725	12568	13992	8366	1100	3193	6942	4293	59228	1033
1984	28446	12774	14817	6293	964	2991	4250	3955	61953	1049
1985	29982	12804	15410	4863	933	2632	2257	3565	63873	1050
1986	31691	12678	13946	5953	978	2115	4685	3093	65342	1054
1987	34166	12811	16327	8451	1239	2290	4935	3529	66911	1102
1988	35026	13200	16027	8176	1173	2340	5349	3513	68918	1136
1989	35711	13287	14601	5620	1219	2503	4306	3722	70572	1130
1990	37931	13547	15225	7372	938	2939	5694	3877	71560	1154

Y^B = output of the business sector; Y^G = output of the public sector; G = government consumption; $M-X$ = net import; I^G = public investment; I^D = housing investment; $YX = Y^G + (M-X) - G$; $IX = I^G + I^D$; K^B = capital stock of the business sector; N^B = employment in the business sector.

Data sources: Bank of Israel, Annual Report, various years, Central Bureau of Statistics, Statistical Abstracts of Israel, various years.

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