Habit formation and distributive justice

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Abstract

Two long-lived agents face an infinite stream of pies, each of size one, one in every period. Each agent ranks consumption streams according to their discounted sum of utilities. Preferences are non-standard in the following sense: the periodic utility in period t is a function of current consumption, c_t, as well as of the previous period's consumption, c_{t-1}. We provide alternative sufficient conditions for the existence of a fair and efficient allocation rule. We also explore in detail several examples for which our conditions do not apply, where fair and efficient allocations may fail to exist. This non-existence stems from the fact that habit results in aversion to consumption smoothing. It is because of aversion to consumption smoothing that one can Pareto-improve on any fair allocation, by introducing a small asymmetry.

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1 Introduction

Past experience is instrumental in shaping present preferences. In this paper we investigate the implications of this intertemporal dependence on distributive justice. More specifically, we investigate whether (or under what conditions) it is possible to allocate resources in a fair and efficient manner, given that agents are long-lived and have *reference-dependent preferences*: in each and every period, their preferences take into consideration their past consumption.

To this end, we consider an infinite horizon economy which is populated by two agents who face an infinite stream of size-1 pies and whose preferences are habit-based. In this economy, there is one pie per period and each period's pie can be divided in any way between the agents but cannot be stored for future usage. Given a consumption steam $c^i = \{c^i_t\}$, agent i's corresponding utility is given by the discounted sum $\sum \delta^{t-1} u(c^i_t, c^i_{t-1}) \equiv U(c^i)$, where u is the (periodic) utility function and $\delta \in (0,1)$ is the discount factor. The non-standard aspect of these preferences, therefore, is that the period-t payoff depends both on that period's consumption, c^i_t, as well as on the previous period consumption, c^i_{t-1}.

In this infinite-horizon economy, an allocation is a pair of non-negative sequences $(c^1, c^2) = (\{c^1_t\}_{t=1}^{\infty}, \{c^2_t\}_{t=1}^{\infty})$ such that $c^1_t + c^2_t \leq 1$ for all t.[1] We fix $c^i_0 = \frac{1}{2}$ for both i; that is, in period 0 the agents shared the pie equally, and now an allocation needs to be specified for the remainder of the time horizon, $t = 1, 2, \cdots$. Given that the "initial condition" $c^i_0 = \frac{1}{2}$ is the same for both agents, and given that the agents have the same discount factor and the same utility function, they are completely symmetric. We therefore define a "fair" allocation to be any allocation that provides the agents with the same lifetime payoff—any allocation such that $U(c^1) = U(c^2)$. An allocation is (Pareto) efficient if there does not exist an alternative allocation, say $(\hat{c}^1, \hat{c}^2)$, such

[1] We consider only deterministic consumption streams; therefore, "allocation," by definition, is non-stochastic. We will comment on stochastic allocations in the concluding Section.

that $(U(\hat{c}^1), U(\hat{c}^2)) \gneqq (U(c^1), U(c^2))$.[2,3] We seek to find under what conditions fair and efficient allocations exist, and what are the economic forces that underlie the tension between fairness and efficiency in our setting.

The answer to these questions depends, of course, on the specifics of preferences. With current and previous-period consumption in a generic period denoted by x and c respectively, our central assumption is the following:

Main Assumption : Given a fixed x, $u(x, c)$ is (weakly) decreasing in c.[4]

The Main Assumption leaves, informally speaking, two "degrees of freedom": it does not say anything about the effect of x on u and does not say anything about the cross-effects between x and c. I particular, for the case where u is differentiable, the Main Assumption only requires $u_2 \leq 0$, but imposes no restrictions on u_1 and $u_{12}(= u_{21})$. Before we proceed, several motivating examples are in place. They serve to illustrate that the Main Assumption is really what we focus on, and that otherwise the utility u is allowed to behave in qualitatively different ways.

Example 1: Desirable consumption. Suppose that current consumption is always desirable ($u_1 > 0$ in the differentiable case), but the degree of its desirability is decreasing in habit ($u_{12} < 0$ in that case). Therefore, if the agent ate "a lot" in period $t-1$ then he will need to eat "a whole lot" in period t in order to reach a high level of utility. For instance, think of taking a vacation in the following two scenarios: (i) you have not gone on a vacation for more than a year, and (ii) you have just returned

[2] $(a, b) \gneqq (a', b')$ if and only if $(a, b) \neq (a', b')$ and $x \geq x'$ for each $x \in \{a, b\}$.

[3] Since there is no uncertainty in our economy, this notion of ("ex ante") efficiency is equivalent to "ex post" efficiency; namely, to the requirement that no Pareto improvement (in terms on continuation payoffs) would exist conditional on every initial history $((c_1^1, c_1^2), \cdots, (c_T^1, c_T^2))$.

[4] Of course, the Main Assumption does not exclude the trivial case of habit-freeness; namely, that $u(x, c) \equiv v(x)$ for some v. Our main result, Theorem 4 below, spells out a condition that excludes this triviality.

from a trip to Hawaii, which was your third vacation in the past year. It may very well be that the extra vacation is valuable even in scenario (*ii*), but it is natural to assume that it is much more appreciated in scenario (*i*). Consequently, for reaching a given utility-level, the vacation from scenario (*ii*) needs to be much nicer than a one from scenario (*i*).

Example 1 describes the phenomenon of getting used to high standard, or "becoming spoiled." Example 2 below describes a different phenomenon.

Example 2: Undesirable consumption. Consider the case that high amounts of previous consumption make current consumption undesirable. Here, as opposed to Example 1, if the agent consumed a lot yesterday, consuming today is painful. For instance, think of alcohol consumption: if the agent got heavily drunk on Saturday night, he would certainly not want to have liquor in Sunday's brunch.

Finally, the following example considers the case that desirability of current consumption can be either positive or negative, depending on habit.

Example 3: Non-monotonic desirability. Consider a similar story to the one from Example 2, except that this time the good we consider is some extremely garlicky dish, not alcohol. It may very well be that current marginal utility—viewed as a function of today's consumption—is positive at first, monotonically decreasing, and negative from a certain point onwards. Our Main Assumption is consistent with such preferences too (e.g., the "tipping point" from which onwards consumption becomes undesirable is monotonically decreasing in habit).

Our Main Assumption, therefore, manifests itself in qualitatively different environments, and it is the implications of *this* assumption that we seek to understand.

We start by exploring two specifications of utility functions (henceforth, models) that show that fairness and efficiency may be incompatible. The utilities in our two models are $u(x, c) = (1-c)x$ and $u(x, c) = \max\{x-c, 0\}$. We call these specifications the *linear model* and the *piecewise linear model*, respectively.[5]

In the linear model fair and efficient allocation exists if and only if $\delta < \frac{1}{2}$; in the piecewise linear model such allocation does not exist, independent of δ. In either model this non-existence is the result of aversion to consumption smoothing. In fact, the aversion to consumption smoothing is so strong (in either model), that the best consumption path for an agent is a rocky path that swings between high and low consumptions—$(h, l, h, l, \cdots)$. More specifically, the first-best consumption path for an agent in these models is $(1, 0, 1, 0, \cdots)$.[6]

When rocky consumption paths are highly desirable, we can "almost" provide each agent with his first-best consumptions plan: we can give one of the agents the stream $c_*^1 = (h, l, h, l, \cdots)$ and give the other agent the "complement" stream, $c_2^* = (1-h, 1-l, 1-h, 1-l, \cdots)$. If the agents are sufficiently patient, then even c_2^* is better than anything that can be achieved under fairness.[7] Thus, the incompatibility of fairness and efficiency in our economy stems from the combination of (i) patience, and (ii) desirability of rocky consumption paths.

The desirability of rocky consumption paths, in turn, stems from the following

[5]Each model offers an alternative formalization for the idea from Example 1. We are particularly interested in preferences such as the ones from Example 1, but our general results (Theorems 3 and 4 below) apply to a wider class of preferences.

[6]This 0-1 structure may seems extreme. However, it is just an artifact of the (piecewise) linearity of the periodic utilities, and it is not at all essential. For example, consider the following perturbation of the linear model, where the utility is $u(x, c) = (1-c)x^\alpha$, where $\alpha < 1$ and close to one. Under these preferences the stream $(1, 0, 1, 0, \cdots)$ is suboptimal because when $c < 1$ the marginal utility at $x = 0$ is infinity. What is true, however, is that this utility specification, like the one of the linear model, makes rocky streams desirable.

[7]The allocation (c_*^1, c_*^2) is not fair, as $U(c_*^1) > U(c_*^2)$. As δ converges to one the term $(1-\delta)(U(c_*^1) - U(c_*^2))$ converges to zero, but for a fixed $\delta < 1$ the difference in payoffs is not insignificant.

tradeoff. Current consumption has two effects: it provides the agent with utility in the present period, but, at the same time, it makes it harder for him to generate utility in the future, because today's consumption is tomorrow's reference point.[8] We refer to the later effect as the *intertemporal externality.* The connection between this externality and the rocky pattern is that consumption plans such as $(h, l, h, l, \cdots)$ or $(l, h, l, h, \cdots)$ cancel (or, at a minimum, mitigate) the externality's adversarial effect. Thus, we are lead to the conclusion that incompatibility of fairness and efficiency results from the combination of (i) patience, and (ii) intertemporal externality.

This leads us to the conjecture that if the intertemporal externality is not too strong, and if the patience level is not too high, then fair and efficient allocations obtain. This conjecture is correct: the central contribution of this paper is formulating a "bounded intertemporal externality" condition and showing that the combination of this condition with a mild *minimal impatience* condition implies the existence of fair and efficient allocations (see Theorem 4).[9]

It should be emphasized that whereas we devote much of our attention to a detailed examination of the linear and piecewise linear models, which, in turn, are two alternative formalizations of Example 1, our possibility results (Theorems 3 and 4) apply to a larger class of preferences. For example, in Section 7 we consider the possibility of an intertemporal externality which is so strong, that current marginal utility may be negative, provided that yesterday's consumption was at a high enough level. To this end we consider a generalization of the linear model, where periodic utility takes the form $u(x, c) = (1 - Mc)x$, for some constant $M \geq 1$. Unimaginatively, we call this specification the *generalized linear model.* Theorem 4 applies to this model as well. This model, in turn, is a formalization of Example 2.

[8]It may even be the case that current consumption makes it "infinitely difficult" to produce utility tomorrow; see Example 2 above.

[9]Our minimal impatience condition is indeed mild: its only requirement is that $(1, 0, 1, 0, \cdots)$ is weakly better than $(0, 1, 0, 1, \cdots)$. Additionally, we also assume that preferences over consumption streams are continuous.

The rest of the paper is organized as follows. Sections 2 and 3 consider the linear and piecewise linear models, respectively. For either model we solve, as an auxiliary step, its *single-person* version; namely, the problem of a decision maker who faces an infinite stream of size-1 pies and needs to decide on a consumption program. Either single-agent problem is described by a Bellman equation. The two Bellman equations are different (as one would expect), but, surprisingly, they admit a common (unique) solution. This coincidence is non-trivial, and we elaborate on it in Section 4. Next, in Section 5, we discuss convexity. Preferences over consumption streams in both the linear and piecewise linear models are not convex. This non-convexity is necessary for the incompatibility of fairness and efficiency, as convexity, when combined with continuity, guarantees the existence of fair and efficient allocations. Section 6 provides the main contribution of the paper—a set of sufficient conditions for the existence of fair and efficient allocations, in which the central condition is *non-trivial habit*; the bounded intertemporal externality requirement is part of that condition. The non-trivial habit condition is independent of convexity. Section 7 considers the generalized linear model. Section 8 concludes. Discussion about the model and its relation to the existing literature is in that Section. An appendix collects several proofs.

2 The linear model

We start by considering the case $u(x, c) = (1 - c)x$. Call this the *linear model*.

Theorem 1. *Assume the linear model. There exists a fair and efficient allocation if and only if* $\delta < \frac{1}{2}$.

To prove this theorem, we start by formulating and solving an auxiliary problem. Consider the liner model with a single agent. Namely, a single agent is facing an infinite sequence of size-one pies, and he needs to decide how much to consume in every period. His periodic utility function, defined on that period's consumption, x, is αx, where $1-\alpha$ is the consumption in the previous period. The Bellman equation for this problem is:

$$V(\alpha) = \max_{x\in[0,1]} \ \alpha x + \delta V(1-x). \tag{1}$$

Let $V^*(\alpha) \equiv \max\{\frac{\delta}{1-\delta^2}, \alpha + \frac{\delta^2}{1-\delta^2}\}$.

Lemma 1. *V^* is the unique solution to (1).*

We can now ready to turn to the 2-agent model. The "only if" direction of Theorem 1 is proved via the following lemmas. The "if" direction will follow from Theorem 4 (we therefore omit its proof here).

Lemma 2. *Suppose that $\delta \geq \frac{1}{2}$. Consider a fair allocation in the linear model. Then the corresponding value for each agent is at most $\frac{\delta}{1-\delta^2}$.*

Proof. Assume by contradiction that the (common) value is strictly greater than $\frac{\delta}{1-\delta^2}$. Wlog, suppose that the share of agent 1 from today's pie, say x, is weakly smaller than one half. We therefore have $\frac{x}{2} + \delta W > \frac{\delta}{1-\delta^2}$, where W is agent 1's continuation value. Since $\delta \geq \frac{1}{2}$, it follows that $\delta(x+W) > \frac{\delta}{1-\delta^2}$, or $W > \frac{1}{1-\delta^2} - x$.

On the other hand, $W \leq V^*(1-x)$, where V^* is the value function from the single agent problem (since the agent consumes x today, his marginal utility tomorrow will be $1-x$). Now, since $x \leq \frac{1}{2}$, $V^*(1-x) = 1-x+\frac{\delta^2}{1-\delta^2}$. Therefore $1-x+\frac{\delta^2}{1-\delta^2} > \frac{1}{1-\delta^2} - x$, which implies $1 > 1$—a contradiction. $\square$

Lemma 3. *There exists an allocation in the linear model that gives one agent the value $\frac{\delta}{1-\delta^2}$ and gives the other agent a value which is strictly larger than $\frac{\delta}{1-\delta^2}$.*

Proof. Consider the following allocation: give the entire pie to agent 1 today, give the entire pie to agent 2 tomorrow, and let them "alternate roles" forever onwards: each receives the entire pie every other period. Clearly, the value for agent 2 is exactly $\frac{\delta}{1-\delta^2}$. For agent 1 it is $\frac{1}{2} + \frac{\delta^2}{1-\delta^2} > \frac{\delta}{1-\delta^2}$. $\square$

3 The piecewise linear model

Here we consider the utility $u(x, c) = \max\{x - c, 0\}$. Call this the *piecewise linear model.*[10] As was the case with the linear model, we start by considering the single-agent version of the problem. Its Bellman equation is:

$$V(c) = \max_{x\in[0,1]}\ \max\{x - c, 0\} + \delta V(x). \tag{2}$$

Let $V^{**}(c) \equiv \max\{-c + \frac{1}{1-\delta^2}, \frac{\delta}{1-\delta^2}\}$.

Lemma 4. *V^{**} is the unique solution to (2).*

Armed with the closed-form solution to the single-agent problem, we can turn back to the 2-person case.

Lemma 5. *Consider a symmetric allocation rule in the piecewise linear model. Then the corresponding value for each agent is at most $\frac{\delta}{1-\delta^2}$.*

Proof. Wlog, suppose that agent 1's share of the first period pie is weakly below on half. This means that his value (and therefore the value of the other agent, because of symmetry) is no greater than $\delta V^{**}(0) = \frac{\delta}{1-\delta}$. □

We are now ready to state and prove our next main result.

Theorem 2. *There does not exist a fair and efficient allocation in the piecewise linear model.*

Proof of Theorem 2: Note that the allocation which is described in Lemma 3 gives one agent the value $\frac{\delta}{1-\delta^2}$ and gives the other agent a strictly higher value. Therefore, the proof follows form Lemma 5. □

[10] This specification can be seen as an extreme instance of the following *loss-aversion* utility:

$$u(x, c) = \begin{cases} x - c & \text{if } x > c \\ l(x - c) & \text{if } c \leq x. \end{cases}$$

where $l \geq 0$ is the loss-aversion parameter. That is, the piecewise linear model corresponds to $l = 0$.

4 An interesting coincidence

In the Bellman equation (2) the state variable is yesterday's consumption, c. In (1), on the other hand, the state variable is current marginal utility, which is $\alpha \equiv 1 - c$. We choose to write (1) in this particular way because it seems intuitive to us to emphasize the role of marginal utility in that model. However, like in the piecewise linear model, in the linear model the state variable can also be taken to be c, due to the 1-1 mapping between c and α. Therefore, the Bellman for the single-agent linear model can be written as:

$$V(c) = \max_{x \in [0,1]} \ (1 - c)x + \delta V(x). \tag{3}$$

As the following result shows, the solution of the Bellman equation from the (single-agent version of the) piecewise linear model also solves the one from the linear model.[11]

Proposition 1. *V^{**} is the unique solution to (3).*

Proof. Given that yesterday's consumption took on the value c, current marginal utility is $1 - c$, and therefore the value for the agent is $V^*(1 - c) = \max\{1 - c + \frac{\delta^2}{1-\delta^2}, \frac{\delta}{1-\delta^2}\} = \max\{-c + \frac{1}{1-\delta^2}, \frac{\delta}{1-\delta^2}\} = V^{**}(c)$. □

The coincidence of the solutions of (2) and (3) has an interesting interpretation in the context of the *integrability problem* from classical demand theory—namely, that knowledge of an agent's demand may not be sufficient for deducing the preferences that underlie this demand.[12] Here what we have is a non-market setting in which an agent who faces an infinite stream of pies needs to decide on a consumption plan, and in which knowledge of the "worth of the problem" (i.e., the value function) as well as knowledge of the optimal policy (the "demand") may not be enough for pinning down the underlying preferences.

[11] One can show that the optimal policy for the agent is the same in either model; for brevity, we omit the proof of this fact.

[12] See, e.g., Mas-Colell et al. (1995), p.75.

One may suspect that the coincidence of the solutions of (2) and (3) is an implication of the (piecewise) linearity of the periodic utility functions (for a fixed c). This is not true: here is an example of a family of simple linear periodic utility functions, with the property that no model characterized by a utility function in this family has a corresponding Bellman equation which is solved by V^{**}.

Given $\lambda \in \mathbb{R}_+$, consider the periodic utility $u(x,c) = x - \lambda c$. The corresponding Bellman is:

$$V(c) = \max_{x \in [0,1]}\ x - \lambda c + \delta V(x). \tag{4}$$

Proposition 2. *There does not exist a λ such that V^{**} solves (4).*

Proof. Assume by contradiction that such a λ exists. Consider the case $c = 1$. The corresponding equation that needs to be satisfied is $\frac{\delta}{1-\delta^2} = \max_{x \in [0,1]}\ x - \lambda + \delta \max\{-x + \frac{1}{1-\delta^2}, \frac{\delta}{1-\delta^2}\}$. Obviously the maximization of the RHS dictates $x = 1$ and therefore $\frac{\delta}{1-\delta^2} = 1 - \lambda + \frac{\delta^2}{1-\delta^2}$, so $\lambda = \frac{1}{1+\delta}$.

Consider, on the other hand, $c = \frac{1}{1+\delta}$. The corresponding equation that needs to be satisfied is $\frac{\delta}{1-\delta^2} = \max_{x \in [0,1]}\ x - \frac{\lambda}{1+\delta} + \delta \max\{-x + \frac{1}{1-\delta^2}, \frac{\delta}{1-\delta^2}\}$. Again, the RHS is maximized at $x = 1$, therefore $\frac{\delta}{1-\delta^2} = 1 - \frac{\lambda}{1+\delta} + \frac{\delta^2}{1-\delta^2}$, and so $\lambda = 1$, a contradiction. $\square$

One may further suspect that the coincidence of the solutions of (2) and (3) is due to the combination of the following two facts: (i) both involve utility which is (piecewise) linear in today's consumption, and (ii) they induce the same ranking on the restricted domain $\{0,1\}^{\mathbb{N}}$. The following example shows that this, too, is not true.

Consider the periodic utility $u(x,c) = (1 - \sqrt{c})x$. The corresponding Bellman is:

$$V(c) = \max_{x \in [0,1]}\ (1 - \sqrt{c})x + \delta V(x). \tag{5}$$

Obviously the preferences induced by this periodic utility on $\{0,1\}^{\mathbb{N}}$ coincide with those of the linear and piecewise linear models. Yet, there are infinitely many δ's such that the (common) value function of these models does not solve (5).

Proposition 3. *If $\delta > \frac{1}{2}$ then V^{**} does not solve* (5).

Proof. Let $\delta > \frac{1}{2}$. Assume by contradiction that it does. Consider $c = \frac{1}{4}$. The expression which is maximized on the RHS is $\frac{1}{2}x + \delta\max\{-x + \frac{1}{1-\delta^2}, \frac{\delta}{1-\delta^2}\}$. If the optimal x is weakly above $\frac{1}{1+\delta}$ then it must be $x = 1$, and the corresponding value is $\frac{1}{2} + \frac{\delta^2}{1-\delta^2}$. On the domain $[0, \frac{1}{1+\delta})$ the objective's derivative with respect to x is $\frac{1}{2} - \delta < 0$, and so if the optimal x belongs to this domain then it is zero. This gives the value $\frac{\delta}{1-\delta^2} < \frac{1}{2} + \frac{\delta^2}{1-\delta^2}$ and therefore the optimal choice is $x = 1$. Therefore, if V^{**} solved the Bellman equation (5) then $\max\{-\frac{1}{4} + \frac{1}{1-\delta^2}, \frac{\delta}{1-\delta^2}\} = \frac{1}{2} + \frac{\delta^2}{1-\delta^2}$, which implies $1 = \frac{3}{4}$, a contradiction. $\square$

5 Convexity

Recall that a preference relation is convex if all its upper contour sets are. Preferences over consumption streams in both the linear and piecewise linear models are not convex.[13] As the following result shows, this lack of convexity is inevitable. The result applies to any economy of the form that was described in Section 1.

Theorem 3. *If preferences over consumption streams are convex and continuous,*[14] *then a fair and efficient allocation exists.*

Proof. Suppose that preferences over consumption streams are convex and continuous. The set of all lifetime utility (i.e., value) allocations, call it $\mathcal{U}$, is a bounded set which is symmetric around the 45°-line. Let $u^* = \max\{u : (u, u) \in \mathcal{U}\}$. By continuity, u^* is well defined. Let $(\{c_t^{1*}\}_{t=1}^{\infty}, \{c_t^{2*}\}_{t=1}^{\infty})$ be an allocation that gives rise to (u^*, u^*). Obviously it is fair; we argue that it is also efficient. If not, then by continuity there exists a strict Pareto improvement: namely, an allocation, $(\{c_t^1\}_{t=1}^{\infty}, \{c_t^2\}_{t=1}^{\infty})$, that

[13] In fact, as the analysis of the single-agent versions of these models show, they are "extremely not convex"—the optimal consumption path for the single agent is $(1, 0, 1, 0, 1, 0, \cdots)$ or $(0, 1, 0, 1, 0, 1, \cdots)$.

[14] A preference relation is continuous if all its upper contour sets closed.

gives rise to the values $(x, y) > (u^*, u^*)$; the allocation $(\{c_t^2\}_{t=1}^\infty, \{c_t^1\}_{t=1}^\infty)$, therefore, gives rise to the values (y, x). Define the allocation $(\{\tilde{c}_t\}_{t=1}^\infty, \{\tilde{c}_t\}_{t=1}^\infty)$ by $\tilde{c}_t \equiv \frac{1}{2}c_t^1 + \frac{1}{2}c_t^2$. By convexity, the value of this allocation for either agent is at least $\min\{x, y\} > u^*$, in contradiction to the definition of u^*. $\square$

Note that it does *not* follow from the above proof that if preferences over streams are continuous and convex then it is fair and efficient to give every agent half a pie in every period. It may be the case that sharing it is optimal to utilize, in each given period, less than the whole pie. For example, this is the case with the utility function $u(x, c) = 1 - c + \log(1 + x)$, which induces preference that are both continuous and convex (convexity is easy to see due to the separability in c and x). For these preferences, the optimal consumption at the *single-agent* version of the problem is independent of c and is given by $x^* = \min\{\frac{1-\delta}{\delta}, 1\}$.[15] Therefore, if $\delta > \frac{2}{3}$, then it is optimal to split equally among the agent a smaller-then-one pie-slice.

Apart from its natural interpretation in the context of classical consumer theory ("averages are better than extremes"), the role of convexity in economic theory is to a great extant technical: it allows for the application of fixed point theorems, and hence guarantees equilibrium existence. Theorem 3 points out to a novel and somewhat surprising implication of convexity: it makes the existence of fair and efficient allocations (in an economy such as ours) possible, whereas without it their existence is not guaranteed.

[15] The Bellman equation of this problem, $V(c) = \max_{x \in [0,1]} 1 - c + \log(1 + x) + \delta V(x)$, is solved by:

$$V(c) = \begin{cases} \frac{\delta - \log\delta}{1-\delta} - c & \text{if } \delta > \frac{1}{2} \\ 1 - c + \frac{\log 2}{1-\delta} & \text{if } \delta \leq \frac{1}{2}. \end{cases}$$

6 Non-trivial habit

Here we provide an alternative set of sufficient conditions for the existence of fair efficient allocations.

Say that preferences over consumption streams satisfy *minimal impatience* if, given the "initial condition" $c_0 = \frac{1}{2}$, the stream $(1, 0, 1, 0, \cdots)$ is at least as good as the stream $(0, 1, 0, 1 \cdots)$. Alongside this mild conditions, our central condition in this Section is *non-trivial habit.* For its formal introduction, the following notation will be useful. Keeping the notation x and c for current and previous-period consumption, we now add the letter y to denote a generic amount of next-period's consumption. For fixed amount of yesterday's and tomorrow's consumption, namely c and y, consider the function $\Psi(x|c, y) \equiv u(x, c) + \delta u(y, x)$. Say that preferences over consumption streams exhibit *non-trivial habit* (NTH) if the following is true for every $y \in [0, 1]$:

- (i) $\Psi(.|c, y)$ is strictly increasing given any $c \leq \frac{1}{2}$, and
- (ii) $\Psi(.|1, y)$ is weakly decreasing.

NTH says that increasing current consumption has a strictly positive effect on an agent's lifetime payoff if yesterday's consumption was "low," and has a negative effect (at least weakly) if yesterday's consumption was at its maximal possible level. We refer to part (i) of the definition as *bounded intertemporal externality*, meaning that the negative externality of current consumption on the next period utility is overshadowed by its positive immediate reward. Though both parts of NTH's definition are important, it is the bounded intertemporal externality condition that expresses the main economic force that guarantees the existence of fair an efficient allocations. Roughly speaking, the idea is that if the intertemporal externality would have been "very high"—if cutting down on current consumption would always be beneficial—then a race to the bottom would ensue, which would drive the consumption of both agents to zero in each and every period—a clearly Pareto-inferior outcome.

We can now state our next result.

Theorem 4. *Suppose that preferences over consumption streams are continuous and satisfy minimal impatience. Suppose further that the non-trivial habit condition holds. Then a fair and efficient allocation exists.*

In the proof of the theorem, we make use of the following lemma.

Lemma 6. *(Free disposal) Suppose that preferences over consumption streams are continuous. Let c be a consumption stream such that $v \equiv U(c) > U(\boldsymbol{0})$ and let $v' \in (U(\boldsymbol{0}), v)$. Then there is a stream $c' \leq c$ such that $U(c') = v'$.*[16]

Note that "free disposal" refers both to utilities and resources: the conditions specified in the lemma guarantee that it is possible to dispose of resources in a way that results in utility reduction.

Proof. Make the aforementioned assumptions. Let c be such that $U(c) = v$ and let $v' \in (U(\mathbf{0}), v)$. For each $\lambda \in [0, 1]$, look at the "shrunken stream" λc. Let $L \equiv \{\lambda \in [0, 1] : U(\lambda c) \leq U(\mathbf{0})\}$ and let $H \equiv \{\lambda \in [0, 1] : U(\lambda c) \geq U(\mathbf{0})\}$. It is enough to prove that $L \cap H \neq \emptyset$. Note that this is indeed the case, because both of these sets are non-empty ($0 \in L$, $1 \in H$), closed (by continuity), and $L \cup H = [0, 1]$; since the unit interval cannot be represented as a disjoint union of two nonempty closed sets, we are done. □

Proof of Theorem 4: Make the aforementioned assumptions. Let π denote the value for an agent (either agent) in a Pareto-best fair allocation: a fair allocation that is not Pareto-dominated by any other fair allocation.[17] Assume by contradiction that there is a Pareto improvement on (π, π)—namely, an alternative allocation that, wlog, gives the agents the values (v_1, v_2) where $v_1 > v_2 \geq \pi$. Let $c = (c^1, c^2)$ be an allocation that generates (v_1, v_2).

Let $t = \inf\{s : s \geq 1, c^1_s > 0\}$. If $t = \infty$ then not giving any pie to any agent

[16] $c' \leq c \Leftrightarrow c'_t \leq c_t$ for all t.

[17] It is obvious that π is well-defined, because of continuity. Geometrically, (π, π) is the in unique intersection point of the 45°-line with the Pareto frontier.

in any period Pareto improves on (π, π), which is impossible, by the definition of π. Therefore, $t < \infty$. If $t = 1$ then shifting a small $\epsilon > 0$ from agent 1's first-period consumption to agent 2 results in the values (v_1', v_2'), where $v_1' > v_2' > \pi$ (this follows from continuity and non-trivial habit). Since $v_1' > v_2' > \pi \geq U(\mathbf{0})$ it follows from Lemma 6 that (v_2', v_2') is feasible, in contradiction to the definition of π. Therefore $c_1^1 = 0$ and by Pareto optimality and non-trivial habit, $c_1^2 = 1$. By the same argument, $(c_2^1, c_2^2) = (1, 0)$; continuing in this fashion we conclude that $c^1 = (0, 1, 0, 1, \cdots)$ and $c^2 = (1, 0, 1, 0, \cdots)$. By minimal impatience, $v_1 \leq v_2$, a contradiction. $\square$

There are preferences for which Theorem 4 applies but Theorem 3 does not. For example, this is the case with the preferences of the linear model when $\delta \in (\frac{1}{3}, \frac{1}{2})$. In other words, NTH does not imply convexity. Convexity, in turn, does not imply NTH, as clearly seen in the case where $u(x, c)$ does not depend on c.

It is worth noting that checking for NTH is typically easier than checking whether preferences over consumption streams are convex—the former requires a verification of a two-term inequality while verification of the later involves infinite-dimensional objects. Finally, NTH, as opposed to convexity, highlights the productive rule of impatience—if (u, δ) are such that the bounded intertemporal condition is satisfied, then so are (u, γ), for every $\gamma \in (0, \delta)$.

7 The generalized linear model

Up until now we assumed that marginal utility is always non-negative. In this Section we consider the possibility of extreme intertemporal externality, where high levels of yesterday's consumption can make present consumption undesirable. We capture this phenomenon in the following generalization of the linear model, where the utility takes the form $u(x, c) = (1 - Mc)x$, where $M \geq 1$. This is the *generalized linear model.*

The following results are analogous to the ones that were derived in Section 2 and are therefore brought without accompanying discussion.

The Bellman equation of the single agent problem is:

$$V(c) = \max_{x\in[0,1]}\ (1 - cM)x + \delta V(x). \tag{6}$$

Let $V_M^{**}(c) \equiv \max\{-cM + \frac{1}{1-\delta^2}, \frac{\delta}{1-\delta^2}\}$.

Lemma 7. *$V_M^{**}(c)$ is the unique solution to (6).*

The following result is a generalization of the "only if" part of Theorem 1 (the latter is obtained by setting $M = 1$).

Theorem 5. *Assume the generalized linear model, where the parameter M satisfies $M < 2$. If $\frac{2}{M} - 1 > \delta \geq 1 - \frac{M}{2}$, then there does not exist a fair and efficient allocation.*

Note that $M < 2$ is necessary for guaranteeing that the theorem's condition is not void. In the other direction, we have the following:

Theorem 6. *Assume the generalized linear model, where the parameter M satisfies $M < 2$. If $\delta < \frac{1}{M} - \frac{1}{2}$, then a fair and efficient allocation exists.*

Proof. Make the aforementioned assumptions. We will verify that minimal impatience and NTH both hold. Note that, in general, minimal impatience is equivalent to

$$u(1,\frac{1}{2}) + \frac{\delta u(0,1)}{1-\delta^2} + \frac{\delta^2 u(1,0)}{1-\delta^2} \geq u(0,\frac{1}{2}) + \frac{\delta u(1,0)}{1-\delta^2} + \frac{\delta^2 u(0,1)}{1-\delta^2},$$

or

$$(1+\delta)[u(1,\frac{1}{2}) - u(0,\frac{1}{2})] \geq \delta[u(1,0) - u(0,1)].$$

In the case of the linear model, this becomes $(1+\delta)(1 - \frac{M}{2}) \geq \delta$. This is equivalent to $\delta \leq \frac{2}{M} - 1$.

Next, consider NTH. Here, $\Psi(x) = (1 - Mc)x + \delta(1 - Mx)y$. Therefore $\Psi' =$

$(1 - Mc) - \delta My$, which is obviously non-positive when $c = 1$. Also, when $c \leq \frac{1}{2}$ we have that $\Psi' \geq (1 - \frac{M}{2}) - \delta M$, which is strictly positive provided that $\delta < \frac{1}{M} - \frac{1}{2}$.

Thus, by Theorem 4, the existence of fair and efficient allocation is guaranteed if $\delta < \min\{\frac{1}{M} - \frac{1}{2}, \frac{2}{M} - 1\} = \frac{1}{M} - \frac{1}{2}$. The equality is because $M \leq 2$. $\square$

It is interesting to note that the sufficient condition in Theorem 6 highlights the tradeoff between patience on the one hand, and the intertemporal externality on the other hand: the greater is M, the smaller is the maximal allowable δ.[18]

8 Conclusion

We have investigated the co-existence of fairness and efficiency in the presence of habit formation. Habit formation and its economic implications are well-studied in the literature (prominent examples include Abel (1990), Boldrin et al. (2001), Campbell and Cochrane (1999), Christiano et al. (2005), Constantinides (1990), Pollak (1970), Ravn and Schmitt-Grohe (2006), and Sundaresan (1989)). However, we are unaware of any previous work that focuses on the connection between habit formation and distributive justice.

From the single-agent perspective, a work which is related to ours is that of Boyer (1983), who studies a dynamic programming problem where an objective of the form $\sum \delta^{t-1} u_t$ needs to be maximized, and and in which the periodic utility payoff u_t depends on a current consumption vector, x_t, and the corresponding vector of the previous period, x_{t-1}; that is, $u_t = u(x_t, x_{t-1})$ for some function u. However, Boyer assumes that u is strictly concave in (x_t, x_{t-1}). Consequently, his work is significantly different than ours: he focuses on the long-run—namely, the steady state—whereas our analysis revolves, at least in part, around rocky consumption paths. Another

[18] Also, note that the combination of Theorems 5 and 6 does not constitute a characterization of fair and efficient allocations in the generalized linear model, because the lower bound from Theorem 5 exceeds the upper bound from Theorem 6; the two coincide iff $M = 1$.

related work is by Rozen (2010), who axiomatizes *linear* habit formation; the habit formation in our models is not linear in her sense.

The intertemporal link that we have assumed is particularly simple: in every period, only the previous period functions as the reference point.[19] One may suspect that the central role that rocky consumption paths have in our work—and, in particular, their connection to the tension between fairness and efficiency— is not likely to present itself in a more general model. This is a natural concern, and we have no claim that our simple "habit formation technology" allows for a high degree of generality. Having that said, there are two points that we find important to emphasize. First, even in a more general model it may be that fair allocations can be Pareto improved upon via an introduction of a small asymmetry; by shifting consumption, say, from player 1 to player 2, it may be possible to simultaneously increase 2's current payoff and 1's continuation value. That is, the idea that Pareto improvements can be obtained at the expense of fairness *because of an intertemporal externality* (appropriately defined) goes beyond the specific setting we have considered. Second, preferences for rocky paths may naturally be encountered in richer models too, where agents take into account longer histories. For example, consider an agent who likes going to Church exactly once a week: going precisely once a week may be preferable to going every day, and also preferable to never going.

One type of intertemporal link that we did not consider is that of *addiction*—that consumption today *increases* marginal utility tomorrow. Under addiction, higher level of present consumption increase future consumption (see Becker and Murphy (1988)). To account for this phenomenon, one needs a model with at least two goods, say x and y, where increasing addition to x means an increasing path of x-consumption, typically accompanied by decreasing path of y-consumption, and—most importantly—an increasing willingness to pay for x (in terms of y). Our single-good economy is not

[19] This "1-recall" habit is rather common in the literature; see, e.g., Abel (1990), and Alonso-Carrera et al (2005).

the right setup for dealing with this form of habit.[20] However, an central aspect of our work which clearly relates to the phenomenon of addiction is that preferences in the spirit of Example 1 from the Introduction—i.e., preferences such that the agent needs to eat "a whole lot" in order to produce utility if he consumed a lot in the previous period—are similar to those of addicts, as addicts need higher and higher dozes in order to be satisfied. On the other hand, an important point that differentiates our assumptions from those corresponding to addiction is that an addict cannot live without the thing to which he is addicted, whereas in our economy "memory resets" every two periods, and a fully rational agent will plan a rocky path (and stick to it).

We have restricted our attention to deterministic allocations. In fact, we did not even describe the agents' preferences over lotteries over consumption streams. The restriction to deterministic streams in the domain of preferences is rather standard; two examples of settings in which such a restriction is common are (i) canonical macro economic models (e.g., Stokey, Lucas, and Prescott (1989)) and (ii) models of intergenerational equity (see Asheim (2010) for a thorough review). Had the agents' preferences over such lotteries satisfied a suitable independence axiom, then they would admit a representation by an affine functional (see Kreps (1988), p.52), and, consequently, the coexistence of fairness and efficiency would be trivial: the set of feasible expected utility allocations would be the convex hull of a bounded set which is symmetric with respect to the 45° line, hence its Pareto frontier would intersect that line. This argument would not go through without independence. With independence, the message of our results is that randomization has a particular social value—it makes fairness and efficiency compatible in environments where they are otherwise not compatible.[21]

[20]For more on addiction, see Gul and Pesendorfer (2007) and the references therein.

[21]For more on the social benefits of lotteries, see, for example, Jones (2008), and Prescott and Townsend (1984). A more recent contribution regarding the benefits of lotteries appears in Hart and Nisan (2012) and Hart and Reny (2012), who show that in an auction environment with multiple items, it may be the case that the revenue-maximizing mechanism is necessarily stochastic.

Our main sufficient condition for the existence of fair and efficient allocations is non-trivial habit. It indicates that impatience has a potential of allowing desirable social outcomes; since the difficulty to reconcile fairness and efficiency stems from an intertemporal conflict, discounting the future's importance results in mitigating this difficulty. This runs against the well-rooted idea, that patience is typically "good."[22] In fact, as is exemplified most clearly in our linear model (see Theorem 1 above), time preferences play a non-trivial role in our analysis, in the sense of the following non-monotonicity: for low values of δ fairness and efficiency co-exist, for higher values of δ they do not, but the discrepancy between fairness and efficiency vanishes as δ converges to one.

Appendix

Proof of Lemma 1: It is easy to see that the RHS of (1) satisfies Blackwell's sufficient conditions for contraction,[23] hence a unique solution exists. It is enough to verify that V^* is a solution. Plugging V^* into the RHS of (1) we obtain that it is

$$\max_{x\in[0,1]}\ \alpha x + \delta \times \{\max[\frac{\delta}{1-\delta^2}, 1 - x + \frac{\delta^2}{1-\delta^2}]\}. \tag{7}$$

We will now solve the maximization program (7). We consider three cases.

Case 1: $\alpha \leq \frac{\delta}{1+\delta}$. Let x^* be a maximizer of the above equation. If $\max[\frac{\delta}{1-\delta^2}, 1 - x^* + \frac{\delta^2}{1-\delta^2}] = \frac{\delta}{1-\delta^2}$, then the corresponding values is $\alpha x^* + \frac{\delta^2}{1-\delta^2} \leq \alpha + \frac{\delta^2}{1-\delta^2} < \frac{\delta}{1-\delta^2}$. However, note that by choosing $x = 0$, the agent obtains the value $\frac{\delta}{1-\delta^2}$. Therefore, the maximizer x^* must be such that $\frac{\delta}{1-\delta^2} < 1 - x^* + \frac{\delta^2}{1-\delta^2}$ and the corresponding value is $\alpha x^* + \delta\{1 - x^* + \frac{\delta^2}{1-\delta^2}\}$. Note that the derivative with respect to x^* is

[22] Two clear examples are the following: (i) in the realm of repeated games, desirable outcomes are, in many cases, achievable by patient players but beyond the reach of impatient ones (e.g., cooperation in the infinitely repeated Prisoner's Dilemma); (ii) in the realm of economic growth, patience is essential, as without it there would be no saving (and no growth).

[23] See Stokey, Lucas, and Prescott (1989), p.54.

$\alpha - \delta < \frac{\delta}{1+\delta} - \delta < 0$, hence $x^* = 0$. Therefore, the maximization's value is $\frac{\delta}{1-\delta^2}$.

Case 2: $\frac{\delta}{1+\delta} < \alpha < \delta$. Again, let x^* be a maximizer. If $\max[\frac{\delta}{1-\delta^2}, 1 - x^* + \frac{\delta^2}{1-\delta^2}] = 1 - x^* + \frac{\delta^2}{1-\delta^2}$, then the value is $\alpha x^* + \delta\{1 - x^* + \frac{\delta^2}{1-\delta^2}\}$. Note that the derivative with respect to x^* is $\alpha - \delta < \delta - \delta = 0$, so $x^* = 0$ and the value is $\frac{\delta}{1-\delta^2}$. However, by setting $x = 1$ the agent can guarantee to himself $\alpha + \frac{\delta^2}{1-\delta^2} > \frac{\delta}{1-\delta^2}$. Therefore $\frac{\delta}{1-\delta^2} > 1 - x^* + \frac{\delta^2}{1-\delta^2}$, hence the objective assumes the form $\alpha x + \frac{\delta^2}{1-\delta^2}$, which is maximized at $x = x^* = 1$, giving rise to the value $\alpha + \frac{\delta^2}{1-\delta^2}$.

Case 3: $\alpha \geq \delta$. If $\frac{\delta}{1-\delta^2} > 1 - x^* + \frac{\delta^2}{1-\delta^2}$ then the maximum is at $x^* = 1$ and the value is $\alpha + \frac{\delta^2}{1-\delta^2}$. Otherwise, the objective assumes the form $\alpha x^* + \delta\{1 - x^* + \frac{\delta^2}{1-\delta^2}\}$, which is nondecreasing in x^*. Since the objective is continuous in x, the value is $\alpha + \frac{\delta^2}{1-\delta^2}$.

The analysis of the three cases show that the maximization's value is $\frac{\delta}{1-\delta^2}$ if $\alpha \leq \frac{\delta}{1+\delta}$ and $\alpha + \frac{\delta^2}{1-\delta^2}$ otherwise. In other words, it is $V^*(\alpha)$. $\square$

Proof of Lemma 4: It is easy to see that the RHS of (2) satisfies Blackwell's sufficient conditions for contraction, so (2) has a unique solution; it is therefore enough to check that V^{**} is a solution.

Consider then the maximization of $\max\{x - c, 0\} + \delta\max\{-x + \frac{1}{1-\delta^2}, \frac{\delta}{1-\delta^2}\}$ over $x \in [0, 1]$. Divide the domain over which we optimize to $[0, \frac{1}{1+\delta}]$ and $(\frac{1}{1+\delta}, 1]$.

Case 1: $c > \frac{1}{1+\delta}$. On the first domain the (namely $[0, \frac{1}{1+\delta}]$) the objective becomes $\delta(-x + \frac{1}{1-\delta^2})$ so clearly the best point on the first domain is $x = 0$. On the second domain the objective is $\max\{x - c, 0\} + \frac{\delta^2}{1-\delta^2}$, so on this domain $x = 1$ is optimal. The overall optimality of $x = 0$ follows from $c > \frac{1}{1+\delta} \Rightarrow \frac{\delta}{1-\delta^2} > 1 - c + \frac{\delta^2}{1-\delta^2}$. Therefore, when $c > \frac{1}{1+\delta}$ it is the case that $\max_x\{\max\{x - c, 0\} + \delta\max\{-x + \frac{1}{1-\delta^2}, \frac{\delta}{1-\delta^2}\}\} = \frac{\delta}{1-\delta^2}$.

Case 2: $c < \frac{1}{1+\delta}$. On $[0, \frac{1}{1+\delta}]$ the objective becomes $\max\{x - c, 0\} + \delta(-x + \frac{1}{1-\delta^2})$. On the domain where $x \leq c$ the choice $x = 0$ is clearly optimal and on the complementary domain the objective's derivative with respect to x is $1 - \delta > 0$, so $x = \frac{1}{1+\delta}$ is optimal. The choice $x = 0$ gives the objective the value $\frac{\delta}{1-\delta^2}$. The choice $x = \frac{1}{1+\delta}$

cannot be optimal for the entire problem because any additional increase of x—say by Δ—further increases the objective's value by $\Delta(1-\delta)$, and so the maximum is at $x=1$, where the objective's value is $1-c+\frac{\delta^2}{1-\delta^2}=-c+\frac{1}{1-\delta^2}>\frac{\delta}{1-\delta^2}$.

To summarize, the analysis of Case 1 and Case 2 implies $\max\{-c+\frac{1}{1-\delta^2},\frac{\delta}{1-\delta^2}\}=\max_{x\in[0,1]}\ \max\{x-c,0\}+\delta\max\{-x+\frac{1}{1-\delta^2},\frac{\delta}{1-\delta^2}\}$. That is, V^{**} solves (2). $\square$

Lemma 7—Sketch of proof: Case 1: $-Mc+\frac{1}{1-\delta^2}>\frac{\delta}{1-\delta^2}$ (i.e., $c<\frac{1}{M}(\frac{1}{1+\delta})$). We need to show that $-Mc+\frac{1}{1-\delta^2}$ equals the maximum (over x) of $(1-cM)x+\delta\times\max\{-cx+\frac{1}{1-\delta^2},\frac{\delta}{1-\delta^2}\}$. The marginal utility from consumption is positive, hence it is easy to see that the optimal x is $x=1$, which indeed gives rise to the value $1-Mc+\frac{\delta^2}{1-\delta^2}=-Mc+\frac{1}{1-\delta^2}$.

Case 2: $c>\frac{1}{M}(\frac{1}{1+\delta})$. Here we need to show the the maximization of $(1-cM)x+\delta\times\max\{-cx+\frac{1}{1-\delta^2},\frac{\delta}{1-\delta^2}\}$ gives the value $\frac{\delta}{1-\delta^2}$. Since marginal utility is negative it follows that the optimal x must be in the range $[0,\frac{1}{M}(\frac{1}{1+\delta})]$ and it is easy to see that $x=0$ is optimal, which indeed gives the desired value. $\square$

Theorem 5—Sketch of proof: Similarly to the case of the linear model, we will prove that the value for an agent in a fair allocation is bounded by $\frac{\delta}{1-\delta^2}$ and that the "alternating scheme" improves on this. Establishing the latter fact—namely, minimal impatience—is equivalent to establishing $1-\frac{M}{2}+\frac{\delta^2}{1-\delta^2}>\frac{\delta}{1-\delta^2}$. Rearranging this gives $\frac{2}{M}-1>\delta$, which holds by assumption.

To prove that the value of fair allocations cannot exceed $\frac{\delta}{1-\delta^2}$, assume by contradiction that there exists such an allocation whose value is strictly above $\frac{\delta}{1-\delta^2}$. Wlog, suppose that agent 1 receives $x\le\frac{1}{2}$ in the first period; his value is therefore $(1-\frac{M}{2})x+\delta W>\frac{\delta}{1-\delta^2}$, where W is his continuation value. By the lower bound on δ it follows that $\delta(x+W)>\frac{\delta}{1-\delta^2}$, or $W>\frac{1}{1-\delta^2}-x$.

On the other hand, $W\le V_M^{**}(x)=\max\{-xM+\frac{1}{1-\delta^2},\frac{\delta}{1-\delta^2}\}$. If the maximum is $-xM+\frac{1}{1-\delta^2}$ then we obtain $\frac{1}{1-\delta^2}-x<-xM+\frac{1}{1-\delta^2}$, or $M<1$. If the maximum is

$\frac{\delta}{1-\delta^2}$ then we obtain $\frac{1}{1-\delta^2} - x < \frac{\delta}{1-\delta^2}$, or $\frac{1}{2} < \frac{1}{1+\delta} < x \leq \frac{1}{2}$. $\square$

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