Microeconomics MSc

Auctions

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Auctions

- An auction is a mechanism for trading items by means of bidding (offering prices).
- Dates back to 500 BC where Babylonians auctioned of women as wives.
- Position of Emperor of Rome was auctioned off in 193 ad.
- Recently auctions were brought into both everyday use with the Internet (ebay and Google) and governmental arena with the spectrum auctions, bonds and environmental markets.
- Can have the bidders trying to buy an item: Christie's, ebay.
- Can have the bidders trying to sell an item : Procurement, priceline.com.

Rules to Auctions

- **First-Price Sealed-Bid Auction**: Everyone writes down a bid in secret. The person with the highest bid wins the object and pays what he bids.
- **Second-Price Sealed-Bid (Vickrey) Auction:** Everyone writes down a bid in secret. The person with the highest bid wins the object and pays the second highest bid. (used for stamps and by Goethe).
- **English Auction:** The auctioneer starts at a reserve price and increases the price until only one bidder is left.
- **Dutch Auction** (Not Demonstrated): The auctioneer starts at a high price and decreases the price until a bidder accepts the price. (Similar to price-drop.tv)
- **All Pay Auction**: Everyone writes down a bid in secret. The person with the highest bid wins. Everyone pays

Tokyo Fish Auction

Tokyo Flower Auction

Classroom Experiment

- You will be in teams. (Probably 5 teams.)
- Each team will have an independent value for winning drawn uniformly from [0,10].
- Write down your bid in each format: First-Price, Second-Price, and All-pay.
- Be careful that you are not overheard by other teams.
- I will collect the bids and tabulate results.
- I will chose at random one of the auction formats to count each period. I will collect the payments according to the rules and pay the winning team's value in NIS.
- Ties will be broken randomly.
- This will be repeated several times.

Two types of Settings: Common and Private

1. Examples of Common Auctions:
   - Spectrum.
   - Oil Drilling.
2. Examples of Private Auctions:
   - Consumption items.
   - Memorabilia

Strategies with Private values: English Auction

- The English: stay in the auction until either
  - you win
  - or the bid goes higher than your value.
- With a different strategy:
  - either one loses when it is worthwhile to win
  - one wins when it is worthwhile to lose.
- The key to understanding this is to understand that staying in does not affect the price one pays if they win only whether one wins (it does affect others' prices).
- It is best to do this independent of what others do.

Strategies with Private Values: 2nd Price Auctions

- 2nd price similar logic to English auction.
- It is optimal to bid one's value.
- One's bid does not affect the price one pays only whether or not one pays.
- Raising one's bid will cause one to win when it is not worthwhile.
- Your value is 5, the other team bids 6. If you bid 7, you will lose: 5-6=-1.
- Lowering one's bid will cause one to lose when it was worthwhile to win.
- Your value is 5, the other team bids 4. If you bid 3, you won't win and could have won 5-4=1.
- Again it is best to do this independent of what others do.

Example of Strategy

- Values are uniform $[0, 10]$ (equal chance of each). Team B bids its value Vb. Team A receives value Va=5 and has to decide on a bid
- Team A bids its value minus 1. In this case, Team A wins whenever $Vb < 4$ and pays B's value. Expected profit is $4/10 * (5 - 2) = 1.2$
- Team A bids its value. In this case, Team A wins whenever $Vb < 5$ and pays B's value. Expected profit is $5/10 * (5 - 2.5) = 1.25$.
- Team A bids its value plus 1. In this case, Team A wins whenever $Vb < 6$ and pays B's value. Expected profit is $6/10 * (5 - 3) = 1.2$.
- Problem in 1st case, A doesn't win when $4 < Vb < 5$. Problem in 2nd case, A wins when $5 < Vb < 6$.

Strategies with Private Values: First Price

- Strategies in the first-price should shade bid below your value
  - This is because one's bid affects one's price.
  - Bidding your value will earn zero surplus.
  - Shading one's bid lowers the probability of winning, but increases the surplus gained when winning.
- There is a natural trade-off between probability of winning and profit if one wins.
  - If bid is b, value is v, expected profit is $Probwin(b)(v-b)$
  - Derivative of this w.r.t. b yields $Probwin'(b)(v-b) - Probwin(b) = 0$
  - First term is marginal benefit of higher prob of winning.
  - Second term is marginal cost to the profit. (chance of winning time cost if one wins)

First-Price Auction w/ Uniform Distribution

- In FP auction the best strategy depends upon what others do (they affect probwin).
  - If everyone bids zero, then you want to bid close to zero.
  - If everyone bids above 1, you don't want to bid close to zero if your value is 2.
- Equilibrium is
  - all bidders choose a strategy $b(v)$: how much to bid given their value.
  - Given the strategy of others, no one wants to change their strategy (Nash).

First price auction equilibrium

- $b^*(v)$ is an equilibrium such that

$$b^*(v) = argmax_b probwin(b)(v - b)$$

- What is prob of winning given there are only 2 bidders and the other bidder bids $b^*(v)$?
  - $Prob(b^*(v) < b) = Prob(v < b^{-1}(b))$
  - Easiest to see this graphically.
  - What is this prob. if $b^*(v) = av$ and $F(v) = v$ (uniform)?
  - What happens if there are $N - 1$ other bidders?

FP auction equilibrium

- Prob of winning given bid b and $N-1$ others are bidding according to $a * v$ is $\left(\frac{b}{a}\right)^{N-1}$
- A bidder maximizes $\max_b \frac{b}{a}^{(N-1)}(v-b)$
- For $N=2$, we have the first-order condition of . . . . (Independent of a).
- This is maximized at $b = \frac{v}{2}$.
- Exercise, do this for $N=3$.
- Anyone want to guess the formula for any N?

Strategies with Private Values: All Pay

- In the all-pay auction, you should again shade bid below your value.
  - If bid is b, value is v, expected profit is
  - $Probwin(b)v - b$. The natural trade-off is now between probability of winning and cost of bidding.
- This cost is incurred whether you win or not.
  - Derivative of expected profits w.r.t. b yields
  - $Probwin'(b)v - 1 = 0$
  - First term is marginal benefit of higher prob of winning.
  - Second term is marginal cost of increased bid.
- It only makes sense to incur a high cost if the probability of winning is fairly high.
- For low values, bids are shaded much more than with first-price auctions.

All-pay equilibria: Linear soln?

- Prob of winning given bid b and $N-1$ others are bidding according to $a \cdot v$ is $(b/a)^{N-1}$
- A bidder maximizes $Max_b(\frac{b}{a})^{N-1}v - b$
- For $N = 2$, we have a clear problem $Max_b(b/a)v - b$
- It is a corner solution unless $a = v$, which is a contradiction (a is constant and v varies).
- We instead need a slightly different approach.

All-pay auction: equilibrium

- Before the bidder chose a bid given his value.
  - Probability of winning was the chance that the other bidder had a value that would induce a lower bid.
- Now have the bidder choose a 'pretend' value $\widetilde{v}$ given their true value v. Their bid is then $b(\widetilde{v})$.
- Bidders then $\max_{\widetilde{v}} \widetilde{v}v - b(\widetilde{v})$
- An equilibrium would just have the bidder choose $\widetilde{v} = v$. Thus, $b'(v) = v$.
- Integration yields $b(v) = \frac{v^2}{2} + const$. The constant is 0, since someone having $v = 0$ would not to bid.
- What is it for $N > 2$ bidders?

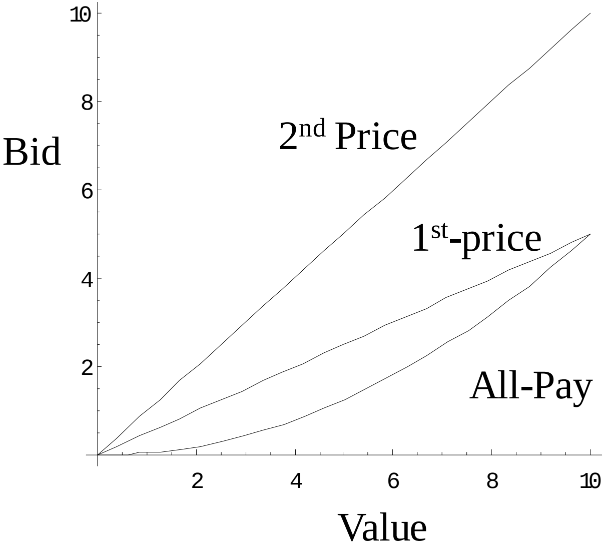
All-pay auction

- May seem like a strange auction to run/study (one which I do research), but. . .
- It is used in charity auctions and from the lab, one can see why. (Losers don't complain so much.)
- Extremely useful modelling tool.
  - Patent Races.
  - Political Campaigns.
  - Technology contests – X-prize, Lindbergh.
  - Procurement contests – Architecture, Next Generation Fighter Jet.
  - Sports contests. (Think of Chelsea, Man U, Arsenal all buying the best players.)

Summary: Strategies with uniform values.

- Values are drawn from 0 to 10 with an equal chance of each amount (like in the experiment). N is the number of bidders.
- 1st-price the equilibrium bid $\frac{N-1}{N}v$ (that is if $v = \$5.50$ and $N = 2$, bid $\$2.75$.
- Dutch auction is the same as the 1st-price.
- 2nd-price, optimal to bid value. English optimal to bid up to one's value.
- All-pay auction, should bid $(N-1) \cdot \left(\frac{v}{10}\right) N \cdot \frac{10}{N}$ (looks complicated but only we can see for low values shade bid more than for high values).

Equilibrium Bid Functions



Which design is best for revenue?

- Basic probability.
- Take N values independently drawn from the uniform distribution $[0, 1]$.
- $N = 1$, the expected value is $1/2$.
- $N = 2$, the expected highest $max\{v_1, v_2\} = 2/3$, the expected lowest $min\{v_1, v_2\} = 1/3$.
- $Max\{v_1, \ldots, v_N\} = N/(N+1)$.
- $Min\{v_1, \ldots., v_N\} = 1/(N+1)$.
- The expected value of the 2nd highest$= (N-1)/(N+1)$
- For a uniform distribution, $[0, a]$, $E[Max\{v_1, \ldots, v_N\}] = \frac{N}{N+1} \cdot a$, etc..

Expected Revenue

- Everyone bids their value. The highest wins it and pays the 2nd highest bid. What is the 2nd highest value?
  - $\frac{N-1}{N+1}$. For $N = 2$, it is just $1/3$.
- For a first-price auction, the highest wins it and pays his bid. He bids $\frac{N-1}{N} \cdot v$.
- $E[\frac{N-1}{N} max\{v_i\}] = \frac{N-1}{N} \cdot E[max\{v_i\}] =$
  - $\frac{N-1}{N} * \frac{N}{N+1} = \frac{N-1}{N+1}$. Same!
- In all-pay, the auctioneer collects money from all bidders. Revenue$=N \int_0^1 b(v)dv = N \int_0^1 \frac{N-1}{N} v^N dv = \frac{N-1}{N+1}$
- They are all the same! Amazing!

Revenue Equivalence: 4 designs

- For private values, there is **revenue equivalence** among all 4 designs: 1st/2nd price, all-pay, English.
- Not only that but all auctions are fully efficient – the buyer who values the object the most winds up buying it.
  - Here total surplus is simply the highest value: $E[max\{v_i\}] = N/(N+1)$.
  - Total surplus=buyers' surplus + seller's surplus.
  - Implies buyers' surpluses are the same.
- If a seller wants to maximize revenue, he can simply use an appropriate minimum bid in any of the designs.
- *Problems happen if:* asymmetry, risk aversion, common values, seller info.

Revenue Equivalence: General

- Two mechanisms that

1. yield the same allocation of the object.
2. have the same surplus for the bidder with the lowest valuation

- Have the following properties

1. The seller's expected revenue is the same.
2. A bidder's expected surplus is the same.
3. Given a bidder's value, his expected surplus is the same.

- *Assumptions needed* are risk neutrality, private and independent values. (Not Symmetry!)