Microeconomics MSc

preference relations

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Guessing Game

- Guess a number 0 to 100.
- The guess closest to $2/3$ the average number wins a prize.
- Ties will be broken randomly.
- Please write your name and your guess on the piece of paper and turn it in to me.
- Don't let others see your guess!

Guessing Game

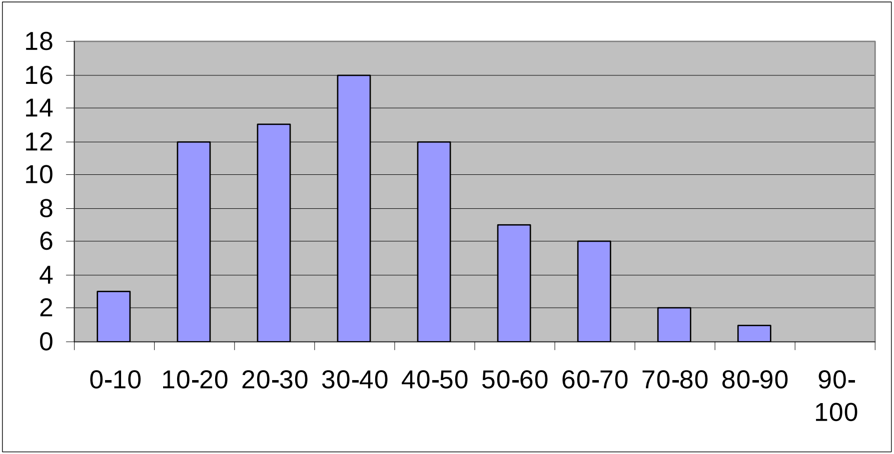

Figure: Graph of guesses (Exeter UGs). Average was 36.7 (winning guess 24.5). Six guesses were above 66.66

Guessing Game

- Guess a number 0 to 100.
- The guess closest to 2/3 the average number wins a prize.

- Your average was . . .
- Wharton average 40
- Caltech UG average was 30 (10% at 0)
- Economics Phds 25
- CEO average also 40.
- What would we expect if everyone is rational and thinks everyone else is rational?
- Now repeat this!

Guessing Game

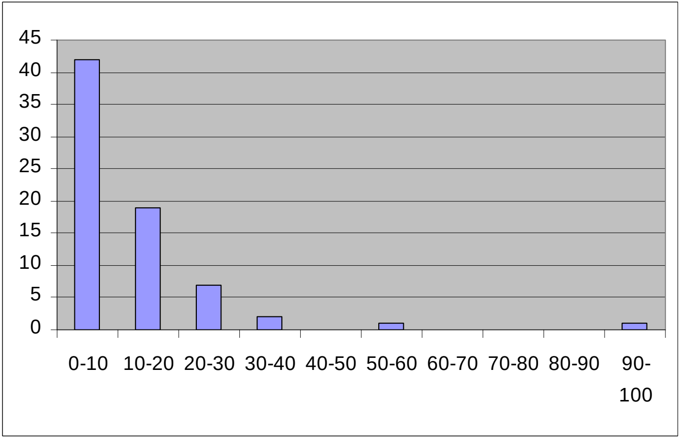

Figure: Exeter UG 2nd round guesses. Average was 12.4 (winning guess 8.3). One guess was above 66.66

Moral of the Story

- Sometimes markets aren't always rational or in "equilibrium". This Guessing game is like a market bubble.
- Sometimes theory based on rationality doesn't predict perfectly.
- You shouldn't ignore theory (even when it is wrong). Otherwise, it may catch you unexpectedly.
- Star Trek: Kirk had several advisors with different views of the world. Spock used logic. Bones emotions. Scotty practicality.
- Here you will learn mostly the rational view.

What is Microeconomics?

- The study choices made by ‘rational’ beings and the interaction of these choices.
- The goal is to build simple models that can be tested either empirically or experimentally (science).
- **Rationality assumption**: people make the best choices they can given their circumstances.
- **Equilibrium:** when everyone is making their best choices (both given their circumstances and what others are doing).
- We will use mathematical tools to predict behavior. Pool players do not make calculations (Friedman).

Goals of the course:

1. To build a solid foundation of microeconomics (tools of the trade).
2. To understand where microeconomic analysis could be useful.

Rationality in Economics

- Behavioral Postulate: A decisionmaker always chooses its most preferred alternative from the set of available alternatives.
- So to model choice we must model decisionmakers' preferences.

Preference Relations

- Comparing two different consumption bundles, x and y from the possibility set $X \subset \mathbb{R}^n_+$:
  - weak preference: x is as at least as preferred as is y.
  - strict preference: x is more preferred than is y.
  - indifference: x is exactly as preferred as is y.
- These are all preference relations.
- They are ordinal relations; *i.e.* they state only the order in which bundles are preferred.

Preference Relations

- $\succ$ denotes strict preference so $x \succ y$ means that bundle x is preferred strictly to bundle y.
- $\sim$ denotes indifference; $x \sim y$ means x and y are equally preferred
- $\succeq$ denotes weak preference; $x \succeq y$ means x is preferred at least as much as is y
- $\not\succeq$ denotes not $x \succsim y$, etc.

Preference Relations: Implications

Each is connected	Can you find these?
$x \sim y \iff x \succsim y$ and $y \succsim x$.	$x \nsim y \iff$?
$x \succsim y \iff x \sim y$ xor $x \succ y$.	$x \not\succsim y \iff$?
$x \succ y \iff x \succsim y$ and $y \not\succsim x$.	$x \nsucc y \iff$?

- Note that $\sim$ and $\succ$ are defined solely by $\succsim$. Hence, we refer to a preference relationship by $\succsim$.
- Show that $x \sim y \implies y \sim x$.

Preference Relations: Implications

Each is connected.

$$x \sim y \iff x \succsim y \text{ and } y \succsim x.$$
$$x \succsim y \iff x \sim y \text{ xor } x \succ y.$$
$$x \succ y \iff x \succsim y \text{ and } y \not\succsim x.$$

Can you find these?

$$x \nsim y \iff x \not\succsim y \text{ or } y \not\succsim x.$$
$$x \not\succsim y \iff x \nsim y \text{ and } x \nsucc y.$$
$$x \nsucc y \iff x \not\succsim y \text{ or } y \succsim x.$$

- Note that $\sim$ and $\succ$ are defined solely by $\succsim$. Hence, we refer to a preference relationship by $\succsim$.
- Show that $x \sim y \implies y \sim x$.

Properties of Preference Relations

Definition

Completeness -For any two bundles x and y in X, it is always possible to make the statement that either $x \succsim y$ or $y \succsim x$.

Definition

Reflexivity - Any bundle x is always at least as preferred as itself; i.e. $x \succsim x$.

- Does Completeness imply Reflexivity?

Applying Completeness

- Completeness allows us to reduce some of our implications.
- Completeness implies $x \not\succsim y \Longrightarrow y \succsim x$
- $x \succ y \Longleftrightarrow x \succsim y$ and $y \not\succsim x$ reduces to $x \succ y \Longleftrightarrow y \not\succsim x$
- $x \not\succ y \Longleftrightarrow x \not\succsim y$ or $y \succsim x$ reduces to $x \not\succ y \Longleftrightarrow y \succsim x$.

Properties of Preference Relations

Definition

(weak) Transitivity: If x is at least as preferred as y, and y is at least as preferred as z, then x is at least as preferred as z; i.e., $x \succsim y$ and $y \succsim z \Longrightarrow x \succsim z$.

Definition

Strong transitivity

$x \succ y$ and $y \succ z \Longrightarrow x \succ z$

Definition

Indifference transitivity

$x \sim y$ and $y \sim z \Longrightarrow x \sim z$

- Does weak transitivity implies indifference transitivity (you can use the implications)?

Rationality

Definition

A preference relationship $\succsim$ is **rational** if it satisfies completeness and transitivity.

When Transitivity may not apply

Natural cycles:Condorcet

- Example of violation: Rock, Paper, Scissors game.
- Voting: Condorcet (1743-94) paradox.
- There are 3 candidates: A, B, C
  - Jim's preferences are $A \succ B \succ C$
  - Sean's preferences are $B \succ C \succ A$
  - Doug's preferences are $C \succ A \succ B$
- Who wins against who?
- With preferences, this is just an aggregation of considerations.
- Tournaments: USA, England, Israel, and Malta. Design one where USA/Israel/England wins.
- Dave wants to make money off his brother's Dean's intransitive preferences. How can he do it?

Homework (week 1).

1. If there is completeness and weak transitivity holds, does $x \not\succ y$ and $y \not\succ z$ imply $x \not\succ z$?
2. Show that weak transitivity implies strong transitivity. (Hint. First show that it implies $x \succsim z$. Next, show that if $z \sim x$, then there is a contradition in that $z \succsim y$.)
3. Show that if weak transitivity holds, $x \succ y$ and $y \sim z \Longrightarrow x \succ z$.
4. Give an example in real life where transitivity doesn't apply.
5. There are 4 contestants for Pop Idol (Chohav Nolad): A,B,C,D. There are 3 voters: Jim, Sean and Doug. Jim's preferences are $A \succ B \succ C \succ D$. Sean's preferences are $B \succ C \succ D \succ A$. Doug's preferences are $C \succ D \succ A \succ B$. Can you design a tournament where D wins?

When Transitivity may not apply

Small differences

- $x \sim y$ if $|x - y| < \varepsilon$.
- Does $x \sim y$ and $y \sim z$ imply $x \sim z$?
- No!
- Does this imply a violation of weak transitivity?
- Where does this apply?
- Think of prices. I don't care if the difference of prices is a penny.

When Transitivity may not apply

Changing preferences

x: Not drinking.
y: One cup of wine per day.
z: Getting wasted.

- I don't drink at the moment, but I may prefer $y \succ x \succ z$.
- Once I drink one cup per day, I may then prefer $z \succ y \succ x$.
- We then have a violition of transitivity.
- Is this correct?
- I don't buy this argument. I think the X needs to be better defined to include the consumption path..

Properties of Preference Relations

Monotonicity: More is better

- Remember $x \in X \subset \mathbb{R}^n_+$. Denote x_i as the quantity of the ith component.

Definition

Weak Monotonicity. If $x_i \geq y_i$ for all i, then $x \succeq y$.

Definition

Strong Monotonicity. If $x_i \geq y_i$ for all i and $x_i > y_i$ for at least one i, then $x \succ y$.

Definition

Local Nonsatiation. Given any x and any $\epsilon > 0$, then there is some bundle y with $|x - y| < \epsilon$ such that $y \succ x$.

- Strong Monotonicity implies Local Nonsatiation. (Not the other way).

Monotonicity

- Does Strong Monotonicity imply Weak Monotonicity?
- Does Weak Monotonicity imply Strong Montonocity?
- Give a counter example.
- One can be indifferent to everything.
- What about $x \succeq y \iff \min_i\{x_i\} \geq \min_i\{y_i\}$?
- Does this satisfy Weak Monotonicity?
- Does it satisfy Strong Monotonicity?
- Take bundles $(2, 1)$ and $(1, 1)$.

Definitions of preference sets

- $B(y) = \{x : x \succeq y\}$ and $B^s(y) = \{x : x \succ y\}$
- $W(y) = \{x : y \succeq x\}$ and $W^s(y) = \{x : y \succ x\}$
- $I(y) = \{x : x \sim y\} = B(y) \cap W(y)$
- These sets are quite general: they don't need to be closed, convex, etc.
- $I(y)$ is an indifference curve (set).
- Prove under weak transitivity indifference curves can't cross (or even touch): $x \nsim y \implies I(x) \cap I(y) = \varnothing$.

Satiation Points

- A bundle strictly preferred to any other is a satiation point or a bliss point.
- What do indifference curves look like for preferences exhibiting satiation?
- What do indifference curves look like if there is a bad instead of a good?

Continuity

Definitions

Continuity: For all y, the sets $B(y) = \{x : x \succeq y\}$ and $W(y) = \{x : x \preceq y\}$ are closed sets.
Closed set: A set X is closed if for any sequence $x_i \in X$, the $\lim_{i\to\infty} x_i \in X$.

Lemma

The intersection of two closed sets is closed. Complement of a closed set is open.

Corollary

Under continuity, $I(z)$ is closed, $B^s(z)$ and $W^s(z)$ are open.

- Continuity also implies there is an indifference curve $I(z)$ between both sets. Why?
- Continuity implies if $y \succ z$ and x is close enough to y, then $x \succ z$. Why? Hint: converge to y.

Homework (week 2).

1. The amount of candy that Joel gets is j, the amount of candy that Anaelle gets is a. Joel prefers more candy to less for himself and less candy to more for Anaelle, that is, $(j_1, a_1) \succsim (j_2, a_2) \iff j_1 \geq j_2$ and $a_1 \leq a_2$.
    1. Are these preferences complete? Why or why not?
    2. Now say that if there is no strict ranking, Joel is indifferent. Draw the sets $B((4,4))$, $W((4,4))$, $I((4,4))$.
    3. Does Joel satisfy transitivity?
    4. Is the set $I((4,4))$ closed?
    5. Does Joel satisfy monotonicity or local non-satiation?

Convexity

Definition

Convexity: if $x \succsim z$ and $y \succsim z$, then $t \cdot x + (1-t) \cdot y \succsim z$ for all $0 < t < 1$. (For two goods it is $(t \cdot x_1 + (1-t) \cdot y_1, t \cdot x_2 + (1-t) \cdot y_2) \succsim z$.)

- For example, x and y are on the same indifference curve. The 50-50 mixture of the bundles x and y, is $(0.5)x + (0.5)y$. This is preferred to x or y.
- Warning. It does not imply $.5x + .5y \succsim y$ when x and y are not on the same indifference curve.
- Remark: There is weak and strong convexity.
- Can you graph convex vs. non-convex indifference curves? Can a satiation point be convex?

Utility

Definition

A utility function $u(x)$ represents a preference relation $\succsim$ if and only if $u(x) \geq u(y) \Longleftrightarrow x \succeq y$.

Example

- Consider the bundles $(4, 1)$, $(2, 3)$ and $(2, 2)$.
- Suppose $(2, 3) \succ (4, 1) \sim (2, 2)$.
- Assign to these bundles any numbers that preserve the preference ordering. Call these numbers utility levels.
- Note: All bundles in an indifference curve have the same utility level.

Utility Theorem

Theorem

Existence Theorem. (Debreu 1960) A preference relation that is complete, transitive and continuous can be represented by a utility function

Fact

There is no unique utility function representation of a preference relation.

Fact

With a discrete possibility set of goods, continuity is satisfied.

- Suppose $U(x_1, x_2) = x_1 x_2$ represents a preference relation.
- Does it represent $(2, 3) \succ (4, 1) \sim (2, 2)$?
- Does $V(x_1, x_2) = U^2 = x_1^2 x_2^2$ also represent the same preferences?

Utility

- $U(x_1, x_2) = x_1 x_2 \implies (2,3) \succ (4,1) \sim (2,2)$.
- How about $W = 2U + 10$?
- W preserves the same order as U and V and so represents the same preferences.

Theorem

If U is a utility function that represents a preference relation and f is a strictly increasing function, then $V = f(U)$ is also a utility function representing those same preferences!

- WARNING: Only preserves ordinal ranking not cardinal. Hence, does not preserve risk aversion

Utility

Theorem

Easy Existence Theorem. A preference relation that is complete, transitive, strongly monotonic, and continuous can be represented by a utility function

Proof.

1. Let $\mathbf{1}$ be a vector of ones i.e.$(1, 1, 1, ..., 1)$
2. Let $B^t(x) = \{t \text{ in } \mathbb{R} : t\mathbf{1} \succeq x\}$ and $W^t(x) = \{t \text{ in } \mathbb{R} : t\mathbf{1} \preceq x\}$
3. $W^t(x)$ is non-empty (has 0). $B^t(x)$ is non-empty (weak monotonicity).
4. Thanks to continuity, there exists t_x in both $B^t(x)$ and $W^t(x)$. Hence, $t_x\mathbf{1} \sim x$.
5. Define $u(x) = t_x$. Show $t_x \geq t_y \iff x \succsim y$.
    - Proof of $t_x \geq t_y \implies x \succsim y$: $t_x \geq t_y \implies t_x\mathbf{1} \succsim t_y\mathbf{1} \implies x \sim t_x\mathbf{1} \succsim t_y\mathbf{1} \sim y \implies x \succsim y$
    - Proof of $x \succsim y \implies t_x \geq t_y$: $x \succsim y \implies t_x\mathbf{1} \sim x \succsim y \sim t_y\mathbf{1} \implies t_x\mathbf{1} \succsim t_y\mathbf{1} \implies t_x \geq t_y$. This last step is by strong monotonicity:
    - Say instead, $t_y > t_x$. If so, by strong monotoniciy t_y $\mathbf{1} \succ t_x\mathbf{1}$**.** This contradicts $t_x\mathbf{1} \succsim t_y\mathbf{1}$**,** hence we must have $t_x \geq t_y$.

Lexicographic preferences

- Look first at good x_1 and then look at x_2.
- For instance, Tall Tom likes to date tall, smart women. His first choice in a partner is by height. If two women are of the same height, he chooses the one with the highest IQ.
- Does such preferences satisfy transitivity?
- Can one represent one by a utility function?
- Does Tom's preferences satisfy all of the necessary assumptions for existence?

Utility w/o Continuity?

- Take the following preferences over age.
- Tank Carter is sent to prison for 5 years for driving with a revoked license (and not showing up to his earlier 6 month sentence to see his brother win the SuperBowl).
- When he is in prison, he prefers to be older since it is closer to his release. When he is outside, he prefers to be younger. (He prefers to be released as well).
- An example of the preference is $t_a \succ t_b$ if and only if
  $t_a, t_b \leq 5$ and $t_a > t_b$
  (or) $t_a, t_b > 5$ and $t_a < t_b$
  (or) $t_a > 5$ and $t_b \leq 5$.
- Is this continuous? (Are B & W closed?) Does the implication hold?
- Is there a utility function that represents it?

Necessary?

Are all conditions necessary?

- Transitivity is necessary. (Real numbers follow transitivity.)
- Completeness and reflexivity we want anyway.
- Monotonic (we know is not necessary).
- Nor is continuity.

Are People Rational?

- We thus far have definited rationality as completeness and transitivity.
- This with continuity allows us to use a utility function to represent preferences.
- Let us take a step back to see if this is actually how people behave.
- In theory, there is no difference between theory and practice. In practice there is. -Yogi Berra

Who is Yogi Berra?

1. An Indian spiritual Leader.

B. A cartoon character.

C. A baseball player.

Relativity

- Yogi Berra is in a restaurant. He is about to order a steak when the waitress tells him the special of the day is spaghetti bolognese. He says "In that case, I'll have the Lasaña". If anyone can find the correct quote let me know.
- This is a common problem Dan Ariely's book. Subscription to the economist.
  1. Internet only \$59.
  2. Print only \$125.
  3. Print and Internet \$125.
- Two solutions:

1. Preferences depend upon the choice set.
2. Choice set contains information.

Homework (week 3).

1. Let us define **one-component convexity**: if $x \succsim z$, $y \succsim z$ and there exists an i such that $x_j = y_j$ for all $j \neq i$, then $t \cdot x + (1-t) \cdot y \succsim z$ for all $0 < t < 1$.
   1. Show that convexity implies one-component convexity.
   2. Give an example where one-component convexity holds but where convexity doesn't.
   3. Give an example that does not satisfy one-component convexity.
2. Brad is indifferent frequently. For him $x \sim y$ if either $x_1 \geq y_1$ and $x_2 \leq y_2$ or if $x_1 \leq y_1$ and $x_2 \geq y_2$. (Otherwise, he prefers more of both goods) Can his preferences be represented by a utility function? If not, which assumption is violated?
3. Tata is never indifferent. She strictly prefers x to y if $x_1 \cdot x_2 > y_1 \cdot y_2$. If $x_1 \cdot x_2 = y_1 \cdot y_2$, she strictly prefers the bundle with more of good 1. Can her preferences be represented by a utility function? If not, which assumption is violated.
4. Using the notation of the proof of the easy existence theorem, show that $t_x > t_y \iff x \succ y$.