Msc Micro I 2009 sample exam. Lecturer: Todd Kaplan.

Exam is closed notes and no cheat (equation) sheet.

Note that this is an example exam to demonstrate the format. Actual exam may vary on questions, topics and difficulty.

Please answer exactly 5 questions. You can choose between 1 and 2, 3 and 4, 5 and 6, 7 and 8, 9 and 10. For instance, answering 1, 4, 6, 8, 10 is valid. Answering 1,2,3,4, 5 is not valid.

1. In class, we assumed that bidders were risk neutral, that is, bidders who paid b for the object that they valued at v would have utility $v-b$. Now assume that bidders are risk averse and have utility of $(v-b)^{\frac{2}{3}}$. Also assume that there are 3 bidders in a first-price auction each with a private value drawn independently from the uniform distribution on $[0,1]$. You are one of these bidders and know that the other bidders are bidding half their value.

(i) What is the probability of you winning with bid b?

(ii) What is your optimal bidding strategy given the other bidders are bidding half their values?

(iii) Is your answer from (ii) also an equilibrium strategy?

(iv) What is the equilibrium in a second price auction? Why?

(v) What is the expected revenue in a second price auction?

(vi) Which has higher revenue, the first-price or second-price auction?

Answer: Bidding higher than one bidder has probability 2b. Higher than both is (2b)^2. The bidder than maximizes (2b)$^2(v-b)^{2/3}$. Solving the FOC yields b=3/4 v. If the others use this strategy, then the new problem is (4b/3)$^2(v-b)^{2/3}$ which also yields b=3/4 v. Hence, it is an equilibrium. In 2nd price they bid their value. (Explaination see slides.) Expected Revenue is (N-1)/(N+1) for 2nd price or 1/2. In FP auction, the expected highest value is 3/4. The expected revenue then is E[max{3/4 v}]=3/4 E[max{v}]=3/4*3/4=9/16. FP auction yields higher revenue.

2. Bill and Ted have gotten into a fight and want to end their friendship. Unfortunately, they jointly own Bill and Ted Surf shop. Bill and Ted each have a private value for the surf shop that is drawn independently from the uniform distribution on $[0,1]$. Bill and Ted decide to run a second price auction where they bid for the shop and donate the revenue to charity. (Assume that the money donated to charity does not affect Bill or Ted's utility.) However, there is a 50% chance that Lawyer Larry will nullify the auction and give each equal ownership. The winner of the auction will not get the money back from the charity.

(i) What will Bill and Ted bid in equilibrium as a function of their private value?

(ii) If Ted has a value v, what is his expected profit in equilibrium (include the 1/2 ownership if Larry nullifies the auction).

In the 2nd price auction, one would bid 1/2 v since they only gain 1/2 a chance of owning the store if they win. If Ted has a value of v then he will win v of the time. His expected profit is then $1/2v \cdot v + 1/2(1/2v) - v \cdot \frac{v}{4} = \frac{1}{4}(v + v^2)$

3. Give an example of the Allais Paradox. Why does it violate VNM expected utility? Show how putting different weights on the probability function can solve this problem.

Answer: See class notes.

4. Give an example of the Ellsberg Paradox. Why does it violate VNM expected utility? Show how putting different weights on the probability function **cannot** solve this problem.

Answer: See class notes.

5. Draw the indifference curves for $u(3) = 2$, $u(2) = 2$, $u(1) = 1$, using an equilateral triangle to represent probabilities.

Answer: Note that u(3) and u(2) are the same. This means they are perfect substitutes and the indifference curves should be lines parallel to the side connecting the corners of 2 and 3.

6. Show independence implies for all L, L', L'', L''' and x, if we have $L \sim L'$ and $L'' \sim L'''$, then we also have $xL+(1-x)L'' \sim xL'+(1-x)L'''$.

$L \sim L' \Rightarrow L \succsim L'$ & $L' \succsim L$ and $L'' \sim L''' \Rightarrow L'' \succsim L'''$ & $L''' \succsim L''$

Independence implies $xL+(1-x)L'' \succsim xL'+(1-x)L''$. We can also use independence to show that $xL'+(1-x)L'' \succsim xL'+(1-x)L'''$ (using 1-x instead of the normal x and L'' instead of L and L''' instead of L' and L' instead of L''). Combining yields $xL + (1-x)L'' \succsim xL' + (1-x)L'''$. Likewise, we can get $xL + (1-x)L'' \precsim xL' + (1-x)L'''$. Together this yields the desired result.

7. A person has $u(w) = e^{2 \cdot w}$. His initial wealth is \$0. He also has a lottery ticket that is worth \$$\ln 7$ with prob. $\frac{1}{2}$ and 0 with prob. $\frac{1}{2}$. What

is his expected utility? What is the lowest price p that he will part with the ticket?

Answer: Expected utility is $0.5e^{2\cdot\ln 7} + 0.5e^{2\cdot 0} = (49+1)/2 = 25$. The lowest price he would be willing to accept is such that $e^{2p} = 25$ or $p = \ln 5 = 1.609$.

8. We have utility of $\ln x$ for $t = 1, 3$ (we don't consume at time $t = 0$ or 2). We receive income of m at time 1. Interest rate is 0% real (Japan/US). Discounting is hyperbolic $(1/(1+t))$. At time zero we decide between consuming at time 1 and time 3. (Notice the change.) How much income would we save? Now say instead, at time 1 we make the same decision. How much would we save then?

Answer: At t=0, we value consuming at t=1 at 1/2 and t=3 at 1/4. Hence, we would choose to consume 2/3 of our income at time t=1. At time 1, the weights change and we value consuming at t=1 at 1 and t=3 at 1/3. Hence, we would choose to consume 3/4 of our income at time t=1. Thus, our preferences aren't stationary.

9. Does $x \nsucc y$ and $y \nsucc z$ imply $x \nsucc z$?

Answer: $x \nsucc y \Longrightarrow y \succsim x$ and $y \nsucc z \Longrightarrow z \succsim y$. If weak transitivity holds, then $z \succsim y$ and $y \succsim x \Longrightarrow z \succsim x \Longrightarrow x \nsucc z$

10. Show that weak transitivity implies strong transitivity. (Hint. First show that it implies $x \succsim z$. Next, show that if $z \sim x$, then there is a contradition in that $z \succsim y$.)

Answer. $x \succ y \Longrightarrow x \succsim y$ and $y \succ z \Longrightarrow y \succsim z$. By weak transitivity, we have $x \succsim z$. The rest of the proof is by contradiction. If $x \sim z$, then $z \sim x \Longrightarrow z \succsim x$, Since $x \succsim y$,by transitivity we then have $z \succsim y$. This contradicts $y \succ z$. Since $x \succsim z \Longrightarrow x \sim z$ xor $x \succ z$ and that we now know that $x \nsim z$, we must then have $x \succ z$. Hence, strong transitivity holds.