Msc Micro I 2009-2010 exam. Lecturer: Todd Kaplan.

Please answer exactly 5 questions. Answer one question from each of sections: A, B, C, and D and answer one additional question from any of the sections A, B, C, or D. For instance, answering 1, 4, 6, 7, 8 is valid. Answering 1, 3, 4, 5, 7 is also valid. Answering 1,2,3,4, 5 is not valid

---

Section A.

1. Show that if weak transitivity holds, then $x \succeq y$ and $y \succ z \Longrightarrow x \succ z$.

Answer1: $y \succ z \Longrightarrow y \succeq z$. By weak transitivity $x \succeq y$ and $y \succeq z \Longrightarrow x \succeq z \Longrightarrow x \sim z$ xor $x \succ z$. By weak transitivity $x \sim z \Longrightarrow z \sim x \Longrightarrow z \succeq x$ and $x \succeq y \Longrightarrow z \succeq y$. This contradicts $y \succ z$ (since $y \succ z \Longrightarrow y \succeq z$ and $z \not\succeq y$).Thus, we cannot have $x \sim z$ and we must have $x \succ z$.

Answer2: Suppose when we have $x \succeq y$ and $y \succ z$, we also have $z \succeq x$. By weak transitivity $z \succeq x$ and $x \succeq y \Longrightarrow z \succeq y$. This contradicts $y \succ z$. Hence, $z \not\succeq x$. If we assume completeness, this implies $x \succ z$.

2. Show independence (and weak transitivity) implies for all L, L', L'', L''' and x, if we have $L \sim L'$ and $L'' \succeq L'''$, then we also have $xL+(1-x)L'' \succeq xL' + (1-x)L'''$.

Answer

$L \sim L' \Longrightarrow L \succeq L'$. By independence we then have $xL+(1-x)L'' \succeq xL'+(1-x)L''$. By relabelling, since $L'' \succeq L'''$ independence also implies $yL'' + (1-y)L' \succeq yL''' + (1-y)L'$. Using $y = 1-x$ and transitivity yields $xL + (1-x)L'' \succeq xL' + (1-x)L'''$.

Section B.

3. There are two firms competing in a market: A and B. There are three levels of output each firm can produce: small, medium or large. These are 10, 20, 30, respectively. Profit of firm 1 is $\pi_1(q_1, q_2) = q_1(55-q_1-q_2)$ and firm 2 is $\pi_2(q_1, q_2) = q_2(55-q_1-q_2)$. Assume they choose levels of output simultaneously. Draw the normal form game. What is the Nash equilibrium? Now assume they choose sequentially, draw the game tree. What is the subgame-perfect equilibrium?

	10	20	30
10	350, 350	250, 500	150, 450
20	500, 250	300, 300	100, 150
30	450, 150	150, 100	−150, −150

Nash equilibrium is $(20, 20)$. I won't draw the game tree, but it is just has the 9 different payments at the end. SP has the first firm choosing 30 and the second firm choosing 10.

4. Two firms are considering entering a market. A firm not entering the market earns 0. If a firm is the only firm in the market, then that firm makes 10 (million shekels). If both firms enter the market, then they earn -5 (million shekels) each. Draw the normal form game and find ALL Nash equilibria.

	not enter	enter
not enter	$0, 0$	$0, 10$
enter	$10, 0$	$-5, -5$

There are two pure strategy N.E. (not enter, enter) and (enter, not enter). If one firm enters with probability p and doesn't enter with probability $1 - p$, then the other firm is indifferent to entering if $0 = (1 - p)10 + p(-5) = 10 - 15p \Longrightarrow p = 2/3$. Hence (1/3 not enter+2/3 enter, 1/3 not enter+2/3 enter) is an equilibrium.

Section C.

5. (i) Draw the indifference curves over lotteries with three possible outcomes: 1, 2, and 3 where $u(1) = 4$, $u(2) = 2$, $u(3) = 1$, using an equilateral triangle to represent probabilities. Indicate the direction of increasing utility.

(ii) Find a $u(1)$, $u(2)$ and $u(3)$ with the same indifference curves as in part (i) but where the direction of increasing utility is reversed.

Answer:

(i) An indifference curve would satisfy $p_1 u(1) + p_2\ u(2) + p_3\ u(3) = c \Longrightarrow p_1 4 + p_2\ 2 + p_3\ 1 = c$. The probabilities must add to 1 so $p_1 + p_2 + p_3 = 1 \Longrightarrow p_3 = 1 - p_1 - p_2$, combining yields $p_1 4 + p_2\ 2 + 1 - p_1 - p_2 = c \Longrightarrow p_1 3 + p_2 = c$. Let us look at the case when $c = 1$. When $p_2 = 0$, $p_1 = 1/3$ and thus $p_3 = 2/3$. When $p_1 = 0$, $p_2 = 1$ and $p_3 = 0$. Thus, an indifference curve is a line from the vertex of 2 to the 1-3 side that is 2/3 the distance from vertex 1 to vertex 3. All indifference curves are parallel to this. They are increasing in the direction of corner 1.

(ii) Changing the sign will reverse the direction. We can also add a constant and keep the same indifference curves. One example is $u(1) = 1$, $u(2) = 3$, $u(3) = 4$.

6. Silly Sam chose a sure chance of \$5000 over a 90% chance of \$6000. He also chose a 45% chance of \$6000 over a 50% chance of \$5000. Show

how Silly Sam cannot have VNM expected utility. Show how by using different weights of probability (with a $w(p)$ function) can explain his choice.

Answer:

Let us normalize $u(0)$ to zero. (If utility is vNM then we can always define a function $v(x) = u(x) - u(0)$.) Sam's first choice implies $u(5000) > .9u(6000)$. The second choice implies $.5u(5000) < .45u(6000)$. Multiplying this by 2, yields $u(5000) < .9u(6000)$.

If we put different weights of the probability. The first inequality is $w(1)u(5000) > w(.9)u(6000)$. The second is $w(.5)u(5000) < w(.45)u(6000)$. If $w(1) = 1, w(.5) = .5, w(.9) = .8, w(.45) = .46$, $u(5000) = 5000$ and $u(6000) = 6000$, then the inequalities are in agreement. The key is we must have $2W(.45) > W(.9)$. If this is the case, we can find a utility function that fits this.

Section D.

7. Indirect utility is given by $v(p_1, p_2, m) = \frac{m}{p_1} + \frac{m}{p_2}$. What are the demands for x_1 and x_2?

Answer: By Roy's identity we have $x_1(p_1, p_2, m) = -\frac{\frac{\partial v(p_1,p_2,m)}{\partial p_1}}{\frac{\partial v(p_1,p_2,m)}{\partial m}} = -\frac{-\frac{m}{(p_1)^2}}{\frac{1}{p_1}+\frac{1}{p_2}} = \frac{mp_2}{p_1(p_2+p_1)}$. Likewise, we have $x_2(p_1, p_2, m) = -\frac{\frac{\partial v(p_1,p_2,m)}{\partial p_2}}{\frac{\partial v(p_1,p_2,m)}{\partial m}} = -\frac{-\frac{m}{(p_2)^2}}{\frac{1}{p_1}+\frac{1}{p_2}} = \frac{mp_1}{p_2(p_2+p_1)}$.

8. Even Steven and odd Todd each receive a natural number, i.e., 0, 1, 2, 3, 4, ... Odd Todd prefers any odd number to an even number. Given that his number is odd (or even), he prefers a smaller numbers. Even Steven prefers any even number to any odd number. Given that his number is even (or odd), he prefers a larger number. Please give a utility function over natural numbers that represents Todd's preferences and one that represents Steven's preferences. If you wish, in your definition of the utility function you can use function $odd(n)$ which equals 1 when n is odd and 0 when n is even.

Answer: Notice that $\frac{1}{n+1}$ is always between 0 and 1 and is decreasing in n. Likewise, $1-\frac{1}{n+1}$ is always between 0 and 1 and is increasing in n. If we have $u_{todd}(n) = \frac{1}{n+1} + odd(n)$ then this represents Todd's preferences. Steven's preferences can be represented by $u_{Steve}(n) = 1 - \frac{1}{n+1} + 1 - odd(n)$.