Bailout Uncertainty, Leverage and Lehman's Collapse

Alex Cukierman[1], Yehuda Izhakian[2] and Yossi Spiegel[3]

[1]Berglas School of Economics
Tel Aviv University

[2]The Jerusalem School of Business Administration
The Hebrew University of Jerusalem

[3]Recanati Graduate School of Business Administration
Tel Aviv University

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Goals

- Model the impact of bailout policy on the volume of leverage and likelihood of a crisis
- Analyze the impact of a change in perceptions about government bailout policy on financial markets and the real economy (Lehman's collapse)
- Analyze the impact of expansionary monetary policy on leverage and risk appetite
- Analyze the impact of a change in perceptions about the economy on financial markets and the economy

Outline

- Introduction
- Framework
- Optimizations
- General equilibrium
- Comparative statics
- Conclusions

Chronology of CDS rate around Lehman's collapse (September-October 2008)

Date	Event	CDS Spread
13-14/9		150
15/9	Lehman files for chapter 11	
16-17/9	Paulson suggests TARP to Congress	250
18-19/9		150
22-23/9	Paulson & Bernanke address Congress	450
24-25/9		350
29/9	Congress rejects Tarp proposal	Almost 450
3/10	Amended Tarp approved by Congress	
5-10/10	Aftermath of approval	150

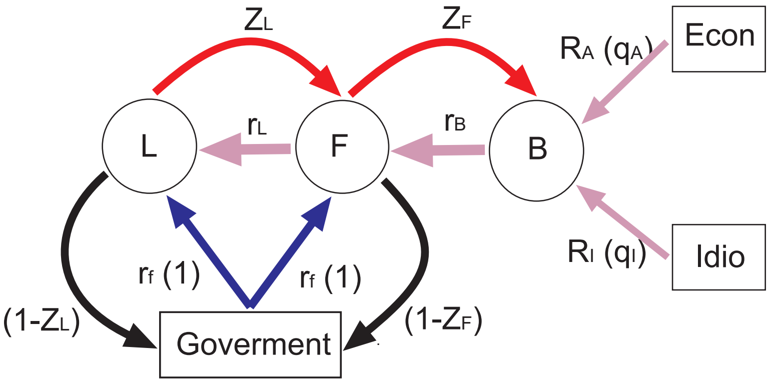
Motivation and mechanism

- Animal spirits (Akerlof Shiller) vs rational agents subject to uncertainty
- The impact of bailout uncertainty on leverage and the role of duration mismatches
- Extended example of a general equilibrium partially microfounded model of financial markets aimed at analyzing the interrelationships between financial markets
- Focus is on the shadow banking system in which long term credit is financed by means of short term liabilities

Recent work

- Meltzer and others, bailout uncertainty
- John Taylor (2009), the impact of low interest rates on the likelihood of a crisis
- Farhi-Tirole (2009), collective moral hazard
- Tirole (2010), illiquidity
- Sieczka, Sornette and Holyst (2010), bankruptcy cascades

Financial flows



Timeline

- 3 periods labeled: $0, 1$ and 2
- All investment decisions are made in period 0
- Once chosen the project size cannot be adjusted

Financing - debt maturity

- Only short term loans are available
- Two periods projects are financed by two consecutive one period loans
- If internal resources and credit refinancing does not suffice a default occurs
- In case of default all equity vanishes

Players

There is a large number of each of the following 3 kinds of players:

- **Borrowers (B)** - entrepreneurs, manufacturers
- **Financial intermediaries (F)** - hedge funds, SIVs and conduits
- **Lenders (L)** - pension funds

Mass: M_B, M_F and M_L

Capital: Each individual player (irrespective of type) possesses one unit of equity capital.
For Fs this unit may be composed of capital and exogenous long term debt.

Borrower

- Each borrower invests in a single long term (two periods) investment project
- The project size, x, is selected to maximizes the borrower's expected utility
- The borrower's utility function is

$$u(W_B) = \begin{cases} W_B & if \quad W_B \geq 0 \\ -F & if \quad W_B < 0, \quad F > 0 \end{cases}$$

- If needed $(x > 1)$ B borrows an additional amount, L_B from F at a gross (one plus) interest rate $r_{Bt}, \quad t = 1, 2$

Financial intermediary

- Borrows an amount L_F from many lenders at r_L
- In each period F lends a fraction z_F of his resources $(1 + L_F)$ to two borrowers at a gross rate r_B
- The remainder, $1 - z_F$, is invested in a risk free asset whose gross rate, r_f, is set by the central bank
- A F chooses L_F and z_F so as to maximize his expected utility from profits, W_F, in each period
- F possesses a quadratic utility function:

$$u\left(W_F\right) = \; W_F - \frac{b}{2} W_F^2, \quad W_F < \frac{1}{b}$$

Lender

- Each lender invests a portion z_L of his resources in a fully diversified portfolio of loans to Fs and a portion $1 - z_L$ in the risk free asset
- Each lender lends to a large number of Fs at r_L. As a consequence his risky portfolio is fully diversified
- Lenders are risk averse characterized by a mean-variance preferences (CARA preferences)

$$u(W_L) = -\frac{1}{\alpha}e^{-\alpha W_L}, \quad \alpha \geq 0$$

Borrower (project) - yield

- Returns on all projects are equally distributed
- A project's gross return in each period is the sum of an aggregate and an individual shock

$$\widetilde{R} = \widetilde{R}_A + \widetilde{R}_I$$

- The aggregate (economy-wide shock) $\widetilde{R}_A$, is binomially distributed:

$$\widetilde{R}_A = \begin{cases} R_A & \text{w.p.} \quad q_A \\ 0 & \text{w.p.} \quad 1 - q_A \end{cases}$$

- The idiosyncratic shock, $\widetilde{R}_I$, is binomially distributed:

$$\widetilde{R}_I = \begin{cases} R_I & \text{w.p.} \quad q_I \\ 0 & \text{w.p.} \quad 1 - q_I \end{cases}$$

Borrower (project) - yield

- $\widetilde{R}_A$ and $\widetilde{R}_I$ are independent across periods and mutually independent within a period
- The idiosyncratic shock, $\widetilde{R}_I$, is independent across projects
- $R_A < R_I$
- The distribution of payoffs is ranked

$$0 < R_A < 1 < \mu_B < R_I < R_A + R_I$$

$$\mu_B \equiv E\widetilde{R}$$

Borrower - default

- Can possibly default in either period 1 (illiquidity) or in period 2 (insolvency)
- A borrower who defaults loses his investment project
- When B defaults the F who lent to him loses the (gross) rate, $r_{Bt}, \quad t = 1, 2$
- A borrower defaults in period 1 if he can't refinance the project
- A borrower defaults in period 2 if his cash flow is smaller than the required debt service

Financial requirements

Financial intermediary - yield

The income from a portfolio consist of two loans $\widetilde{r}_B$:

	Yield	**Probability**
The two borrowers are solvent	r_B	$q_A + (1 - q_A)q_I^2$
One borrower defaults and one is solvent	$\frac{1}{2}r_B$	$2(1 - q_A)(1 - q_I)q_I$
Both borrowers default	0	$(1 - q_A)(1 - q_I)^2$

Financial intermediary - default

- Can possibly default in either period 1 (illiquidity) or in period 2 (insolvency)
- Defaults when at least one of his two borrowers defaults
- His creditors (L) lose the fraction of the (gross) rate, r_L, invested in that particular F provided there are no governmental bailouts.

Financial requirements

Governmental bailout policy

- Government may repay the gross debt owed to lenders by defaulting Fs
- The probability that the debt service of a defaulting F is payed by government (a bailout) is π
- Likelihood of bailout is independent across Fs
- In case of bailout lenders receives the debt service r_L

Lender - yield

The return to a lender on his (fully diversified) portfolio of loans is normally distributed with mean

$$E\left(\{\widetilde{r}_L\}\right) = \left(q_A + (1-q_A)q_I^2 + \pi(1-q_A)\left(1-q_I^2\right)\right) r_L$$

and variance

$$Var\left(\{\widetilde{r}_L\}\right) = q_A(1-q_A)(1-\pi)^2(1-q_I^2)^2 r_L^2$$

Variance of return on risky portfolio is the covariance between two loans.

Equilibrium's interest rates

Proposition

In equilibrium with positive quantities the yields satisfy $0 < r_f < r_L < r_B$

Summary

Player	Index	Leverage	Mass	Income	Expenses
Borrower	B	L_B	M_B	$\widetilde{R}$	r_B
FI	F	L_F	M_F	$\widetilde{r}_B$	r_L
Lender	L	0	M_L	$\widetilde{r}_L$	0

- $\widetilde{R} = \widetilde{R}_A + \widetilde{R}_I$
- Probability of receiving R_A is q_A
- Probability of receiving R_I is q_I
- Probability of bailout is π
- Probability of receiving r_B is determined by q_A and q_I
- Probability of receiving r_L is determined by q_A, q_I and π

Borrower's optimal leverage

- Since Bs payoffs are discrete the probability of default is a step function of L_B
- The optimal leverage is

$$L_B^* \cong \frac{\mu_B + (R_A - 1) r_{B2}^e}{r_{B2}^e (1 + r_{B1}) - \mu_B - r_{B2}^e R_A}$$

Financial intermediary's optimization

- F's wealth is at the end of each period

$$\widetilde{W}_F = (1 + L_F)\left[z_f \widetilde{r}_B + (1 - z_f) r_f\right] - r_L L_F$$

- The distribution of return from two loans, $\widetilde{r}_B$, is

Return	**Probability**	**State**
r_B	γ_F^1	S
$\frac{1}{2} r_B$	γ_F^2	PD
0	$1 - \gamma_F^1 - \gamma_F^2$	D

Financial intermediary's optimization

Proposition

At an optimum with positive leverage F invests all his resources in risky loans to Bs and

$$L_F^*(r_B, r_L) = \frac{(\gamma_F^1 + 0.5\gamma_F^2) r_B - r_L - b\left[\gamma_F^1 r_B (r_B - r_L) + 0.5\gamma_F^2 r_B (0.5 r_B - r_L)\right]}{b\left[\gamma_F^1 (r_B - r_L)^2 + \gamma_F^2 (0.5 r_B - r_L)^2 + (1 - \gamma_F^1 - \gamma_F^2) r_L^2\right]}$$

where

$$\begin{aligned} \gamma_F^1 &= 1 - (1 - q_A)(1 - q_I^2) \\ \gamma_F^2 &= 2(1 - q_A)(1 - q_I) q_I \end{aligned}$$

Financial intermediary's optimization

Proposition

Provided the marginal utility from W_F is positive at twice the value of W_F in the full solvency state (which is the case when b is sufficiently small):

$$\frac{\partial L_F^*}{\partial r_L} < 0, \qquad \frac{\partial L_F^*}{\partial r_B} > 0.$$

Lender's optimization

Proposition

A lender's optimal investment in a fully diversified risky portfolio of loans to Fs satisfies

$$z_L^* = \frac{E\left(\{\widetilde{r}_L\}\right) - r_f}{\alpha Var\left(\{\widetilde{r}_L\}\right)}.$$

General equilibrium in period 1

- General equilibrium of the financial system is characterized by clearing in two credit markets: the market for loans by Fs to Bs and the market for loans by Ls to Fs
- Given the aggregate demand for loans by borrowers and realized rates of return on real investments the market clearing conditions determine r_B and r_L
- In a state of aggregate expansion (E) borrowers' rates of return are

$$R_A + R_I \text{ or } R_A$$

- In a state of aggregate contraction (C) borrowers' rates of return are

$$R_I \text{ or } 0$$

General equilibrium in period 1

- Although equilibrium on financial markets varies depending on whether the economy is in state E or in state C in period 1 the market clearing conditions are qualitatively similar.

General equilibrium in period 1

Expansion

$$q_A(1-q_I)M_B\left\{(1+L_B^*)(1-R_A)+r_{B1}L_B^*\right\} = M_F\left\{W_{F1}+L_F^*(r_{B2},r_{L2})\right\}$$

$$M_F L_F^*(r_{B2},r_{L2}) = \left(\gamma_{LE}^1+\frac{\gamma_{LE}^2}{2}\right) r_{L1} M_L \frac{E\left(\{\widetilde{r}_{L2}\}\right)-r_{f2}}{\alpha Var\left(\{\widetilde{r}_{L2}\}\right)}$$

where W_{F1} is the capital of a solvent F, and γ_{Li}^1 and γ_{Li}^2, $i=E,C$, are messy functions of q_A, q_I and π.

General equilibrium in period 1

- The left hand side (LHS) of the first equation is total demand for loans by non rationned Bs and the RHS is the supply of such loans by Fs that survive into period 1
- The LHS of the second equation is total demand for loans by Fs and the RHS is the supply of such loans by Ls

General equilibrium in period 1

Contraction

$$q_I(1-q_A)M_B\left\{(1+L_B^*)(1-R_I)+r_{B1}L_B^*\right\} = q_I^2 M_F\left\{W_{F1}+L_F(r_{B2},r_{L2})\right\}$$

$$q_I^2 M_F L_F(r_{B2},r_{L2}) = \left(\gamma_{LC}^1+\frac{\gamma_{LC}^2}{2}\right) r_{L1}M_L\frac{E\left(\{\widetilde{r}_{L2}\}\right)-r_{f2}}{\alpha Var\left(\{\widetilde{r}_{L2}\}\right)}$$

Proposition

A decrease in the probability of bailout reduces $\gamma_{Li}^1+\frac{\gamma_{Li}^2}{2}$, $i=E,C$.

General equilibrium in period 0

- There are 6 equilibrium conditions: Period's 0 equilibrium conditions (2 equations), period's 1 forecasted equilibrium condition for the case of expansion (2 equations) and period's 1 forecasted equilibrium condition for the case of contraction (2 equations).
- There is one model consistent equation for expectations about r_{B2}
- Those 7 conditions determine the following 7 endogenous variables: $r_{B1}, r_{L1}, r^E_{B2}, r^E_{L2}, r^C_{B2}, r^C_{L2}, r^e_{B2}$

General equilibrium in period 0

$$M_B L_B^* \equiv M_B \frac{\mu_B + (R_A - 1) r_{B2}^e}{r_{B2}^e (1 + r_{B1}) - \mu_B - r_{B2}^e R_A}$$
$$= M_F \left\{1 + L_F^*(r_{B1}, r_{L1})\right\}$$

$$M_F L_F^*(r_{B1}, r_{L1}) = M_L \frac{E\left(\{\widetilde{r}_{L1}\}\right) - r_{f1}}{\alpha Var\left(\{\widetilde{r}_{L1}\}\right)},$$

Period's 0 forecasts of period 1 equilibrium conditions

In case of aggregate expansion ($\widetilde{R}_A = R_A$)

$$q_A(1-q_I)M_B\left\{(1+L_B^*)(1-R_A)+r_{B1}L_B^*\right\} = M_F\left\{E_0W_{F1}+L_F^*(r_{B2}^E,r_{L2}^E)\right\}$$

$$M_FL_F^*(r_{B2}^E,r_{L2}^E) = \left(\gamma_L^{1E}+\frac{\gamma_L^{2E}}{2}\right)r_{L1}M_L\frac{E\left(\{\widetilde{r}_{L2}\}\right)-r_{f2}}{\alpha Var\left(\{\widetilde{r}_{L2}\}\right)},$$

In case of aggregate contraction ($\widetilde{R}_A = 0$)

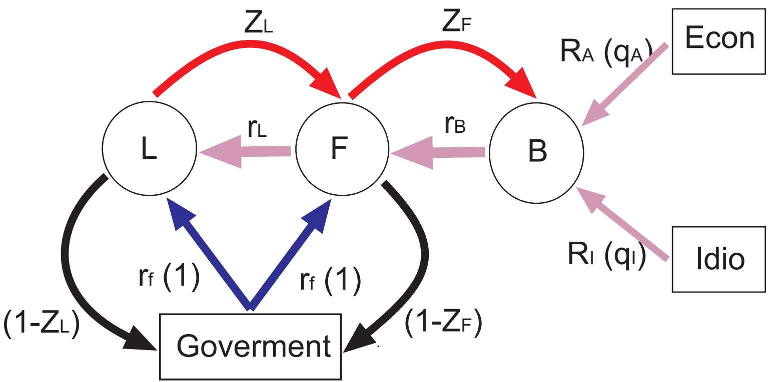
$$q_I(1-q_A)M_B\left\{(1+L_B^*)(1-R_I)+r_{B1}L_B^*\right\} = \gamma_F^{1C}M_F\left\{W_{F1}+L_F(r_{B2}^C,r_{L2}^C)\right\}$$

$$\gamma_F^{1C}M_FL_F(r_{B2}^C,r_{L2}^C) = \left(\gamma_L^{1C}+\frac{\gamma_L^{2C}}{2}\right)r_{L1}M_L\frac{E\left(\{\widetilde{r}_{L2}\}\right)-r_{f2}}{\alpha Var\left(\{\widetilde{r}_{L2}\}\right)}$$

Period's 0 Expectation about r_{B2}

$$r^e_{B2} = q_A(1 - q_I) r^E_{B2} + q_I(1 - q_A) r^C_{B2}$$

Financial flows



Changes in beliefs about bailout policy (period 1)

Following a major indication of a shift in government's bailout policy, like not rescuing Lehman, the (perceived) probability of bailout, π, decreases.

Proposition

A decrease in the perceived probability of bailout reduces $E\left(\{\widetilde{r}_L\}\right)$ *and raises* $Var(\{\widetilde{r}_L\})$. *Provided the difference between* r_{L2} *and* r_{f2} *is smaller than 100 percent, both changes operate to reduce the supply of funds by lenders to financial intermediaries.*

Changes in beliefs about bailout policy (period 1)

Lenders:

- The decrease in perceived π induces a lower expected return and a higher volatility on funds lent to Fs
- The fraction of funds offered as loans to Fs decreases and the fraction invested in the risk free asset increases
- The cost of funds, r_L, to Fs increases inducing deleveraging by Fs
- If perceptions are model consistent the decrease in π also directly reduces the fraction of lenders with positive funds, which induces an additional increase in r_L

Changes in beliefs about bailout policy (period 1)

Financial intermediaries:

- Reduce their leverage, L_F, and the total volume of loans supplied to borrowers
- The cost of funds, r_B, to borrowers increases

Borrowers:

- Borrowers' demand for funds in period 1 is totally insensitive to r_B
- A higher r_B reduces the expected profit for period 2
- Some of the Bs that had expected, as of period 0, to get refinancing in period 1 may not get it

Decrease in the risk free policy rate

- **Lenders:** raise the fraction of their funds supplied to financial intermediaries (reduces r_L)
- **Financial intermediaries:** increase their leverage and supply more funds to borrowers (reduces r_B)
- **Borrowers:** provided the reduction in r_B is sufficiently large, raise their leverage

Change in beliefs about the economy

Proposition

1. *A decrease in either of q_A or of q_I reduces the expected return on the risky portfolios*
2. *Provided q_A and q_I are larger than 0.5 a decrease in either of them raises the variability of the rate of return on the risky portfolios*

The proposition implies that the qualitative effects of more pessimistic expectations about the economy are identical to the impacts of a decrease in the likelihood of governmental bailouts.

Changes in beliefs about idiosyncratic shocks

- Qualitatively similar to comparative statics with respect to the perceived probability of bailouts

The impact of perceived bailout probability (period 0)

Proposition

Provided the difference between r_{L0} and r_{f0} is smaller than 100 percent a higher perceived bailout probability in period 0 is associated with

1. *A higher supply of funds by Ls to Fs and a lower r_{L0}*
2. *More highly leveraged Fs, a higher supply of funds to Bs and a lower r_{B0}*
3. *More highly leveraged Bs*

The impact of risk free rate (period 0)

Proposition

A lower risk free rate in period 0 is associated with

1. *A higher supply of funds by Ls to Fs and a lower* r_{L0}
2. *More highly leveraged Fs, a higher supply of funds to Bs and a lower* r_{B0}
3. *More highly leveraged Bs*

The impact of updating belief on bailout

The combined impacts of an optimistic perception about the probability of bailout in period 0 and of its downward adjustment in period 1:

- A high π_0 leads to an overall expansion of credit in period 0, thus financial intermediaries raise their leverage and so do borrowers
- This makes both Bs and Fs more susceptible to default when π_0 is revised downward in period 1
- The larger the (positive) difference $\pi_0 - \pi_1$, the larger leverage buildup in period 0 and the larger the volume of deleveraging and of defaults in period 1
- The broad conclusion is that bailout uncertainty raises the amplitude of booms and busts

Conclusions

- A decrease in the perceived likelihood of a bailout (lower π) leads to deleveraging in the shadow banking system. This is consistent with the dramatic decrease in SIV's, conduits and hedge funds following Lehman's collapse
- This triggers a general increase in interest rates and an increase in the volume of bankruptcies
- Intensification of pessimistic expectations about the economy (lower q_A and q_I) produces similar qualitative effects

Conclusions

- Expansionary monetary policy (lower r_f) raises leverage, lowers rates and moderates bankruptcies in the short run
- But in the longer run expansionary monetary policy, by encouraging the expansion of short term debt, raises the likelihood of a crisis
- Bailout uncertainty raises the amplitude of booms and busts

Future research

- Use model to investigate how shifting bailout policy from financial institutions to borrowers affects leverage, the volume of defaults and other variables
- An interesting extension of the model will involve allowing financial intermediaries to obtain funds by means of long term (2 periods) as well as short term (one period) loans

Borrower's financial requirements

- Borrower's financial requirements in period 1 are either zero (when realized return is $R_A + R_I$) or, otherwise,

$$FR = (1 + L_B)(1 - \widetilde{R}_{B1}) + r_{B1} L_B$$

- If he survives to period 2 B's terminal wealth is

$$W_B = (1 + L_B)\widetilde{R}_{B2} - FR r_{B1}$$

Borrower's solvency conditions

A borrower is solvent in period 1 iff he obtains the refinancing required to maintain the project till period 2:

$$(1 + L_B)\mu_B \geq FRr_{B1}$$

or equivalently

$$L_B \leq \frac{\mu_B + (\widetilde{R}_{B1} - 1)r_{B2}}{K(\widetilde{R}_{B1}, \mu_B)} \equiv L_B^c(\widetilde{R}_{B1}),$$

where

$$K(\widetilde{R}_{B1}, \mu_B) \equiv r_{B2}(1 + r_{B1}) - \mu_B - r_{B2}\widetilde{R}_{B1}.$$

Borrower's solvency conditions

A borrower is solvent in period 2 iff his cash flow is larger than the required debt service

$$(1 + L_B)\widetilde{R}_{B1} \geq FRr_{B1}$$

or equivalently

$$W_B = A(\widetilde{R}_{B1}, \widetilde{R}_{B2}) - K(\widetilde{R}_{B1}, \widetilde{R}_{B2})L_B \geq 0,$$

where

$$A(\widetilde{R}_{B1}, \widetilde{R}_{B2}) \equiv \widetilde{R}_{B2} + r_{B2}\widetilde{R}_{B1}.$$

▸ Default

Borrower's optimization

An optimal positive level of B's leverage implies

$$-K(\mu_B, \mu_B) > 0.$$

Lemma

Period's 0 $B's$ optimal level of leverage must be one of the following

$$\begin{aligned} L_{B3} &= L_B^c(R_A) - \varepsilon,\ \varepsilon > 0 \textit{ and infinitesimally small} \\ L_{B4} &= L_B^c(0) - \varepsilon \end{aligned}$$

Lemma

Under some additional conditions on W_B for various combinations of $\widetilde{R}_{B1}$ and of $\widetilde{R}_{B2}$ the expected utility of a representative borrower is larger when leverage is at L_{B3} than when leverage is at $L_{B4.}$ implying that L_{B3} is $B's$ optimal leverage. This is more likely to be the case, the larger R_A and the lower $q_A(1-q_I)$.

Financial intermediary's solvency conditions

- F is solvent iff

$$W_F(L_F) = (1+L_F)\widetilde{r}_B - r_L L_F = \widetilde{r}_B + (\widetilde{r}_B - r_L)L_F \geq 0$$

- Since $r_B > r_L$, F is solvent for any L_F when $\widetilde{r}_B = r_B$
- In the other two cases F is solvent only if L_F is sufficiently small:

$$\begin{array}{lll} L_F \leq \frac{0.5r_B}{r_L - 0.5r_B} & \text{when} & \widetilde{r}_B = 0.5r_B \\ L_F = 0 & \text{when} & \widetilde{r}_B = 0 \end{array}$$

- We focus on an equilibrium in which Fs' risk aversion as characterized by b is such that

$$L_F^* > \frac{0.5r_B}{r_L - 0.5r_B}$$

Default