# Optimal Patent Breadth for OLG Economy

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**Abstract**

Existing literature on optimal patent policy and growth has been focused on the infinite horizon economy models. The lack of analogous analysis for an economy of finitely-living consumers motivates the present study. We develop a simple quality ladder model of an overlapping-generations economy in the spirit of the canonical model by Grossman and Helpman (1991). Then we analyze the effect of changes in innovators' marker power - through changes in patent breadth protection - on the stationary R&D investment and welfare. We find that these are determined by the inter-temporal elasticity of substitution (IES). Namely, we find that for plausible values of IES – that is lower than one – weakening innovators market power by narrowing lagging breadth protection, counter – intuitively, has positive effect on R&D investment, but not necessarily on welfare. This crucial role of the IES in patent-policy evaluation is unique to the OLG framework, whereas in the analogous infinite horizon model it has no relevance for the steady state performance.

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## 1. Introduction

The design of optimal patent policy has been central to the research on industry performance, at least from the early 80's[1]. The emergence and prosper of R&D-driven growth literature during 90's however, had called for the implementation of optimal patent policy in a general equilibrium framework. While most of previous studies devoted attention to the infinite horizon frame work established by Grossman and Helpman (1991) and Aghion and Howitt (1992), the present paper contributes to the literature an exploration of optimal patent policy in a model with finite-live consumers.

The four dimensions of patent protection discussed in the literature are patentability requirement, patent length, and breadth. Length of a patent is defined by duration from grant to expiration. Patentability requirements define the minimal size of innovation that grants a patent. Patent breadth is decomposed to leading and lagging breadth. The lagging breadth protection defines the degree on allowed imitation or inferior variant, while the leading breadth defines the right of an innovator over future innovations based on her owns. The present study focuses on minimal patentability requirement and patent breadth protection – both lagging and leading.

For the early R&D - growth models that are based on variety expansion, patent length is the only relevant policy. Optimal patent length for these models was studied (among other) by Judd 1985, Cho and Shy 1993, and more recently by Kwan and Lai (2003)[2]. The canonical quality-ladder models have taken innovation size as exogenous and innovation-effort intensity as endogenous. In these models the patent policy combines infinite length and complete lagging protection, with no leading protection. The lack of leading breadth protection makes the statutory patent length protection ineffective, because once (any) quality improvement was achieved the value of the previous technology deteriorates to zero. In this literature it is also assumed that the diffusion of knowledge embodied in new innovations is perfectly shared by all market participants. That is, an entrant in the R&D sector can freely catch up with the industry-leader's knowledge (productivity) just by observing previous innovations. Under this assumption a weaker lagging breadth protection in this set up, conventionally hinders innovation and

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[1] Jaffe (1999) comprehensively surveys the research on patents and patent policy design, and American patent policy in practice.

[2] See also Helpman (1993) and Barrao and Sala-I-Martin (1997) for analysis of imitation in the context of international trade with leading-innovating economies and following - imitating economies

technological progress, by eroding innovators market power and thereby eroding the inventive to innovate as in Li (2001) in analysis of exogenous innovation size[3].

In later elaborations of the quality-ladder models imitation has been perceived as a necessary, intermediate, step in the process innovation: only through imitation can a firm learn about and improve on the current cutting-edge (state of the art) technology. See for example Aghion et al. (2001) and Mukoyama (2003). In such a setup, an incomplete lagging breadth protection may spurs innovation, by allowing some degree of learning through imitation.

More recent studies have been focused on optimal leading breadth protection in economies of quality-ladder growth. The broader leading breadth protection results in higher market power, and thereby higher markup, due to consolidation of successive innovators through licensing agreements, as proposed by (O'Dowoghue et al. 1998). However, these overlapping property rights may also induce an adverse effect – named "back loading" effect – by delaying the return on R&D investment, which may weaken the incentive to innovate. O'Dowoghue and Zwemuller (2004) study the effect of leading breadth on growth in the canonical quality ladder growth model of Grossman and Helpman (1991). They show how both minimal patentability-requirement and broader leading breadth protection can stimulate R&D effort. They do not provide welfare analysis for these policy implementations, however. Chu (2009) provides a DGE calibration of the blocking effect caused by the leading breadth protection which he finds to be significant for US data. In addition he points at the dynamic distortion caused by monopolistic markup pricing which leads to under investment and accumulation of physical capital (firstly identified by Leither, 1982). This dynamic distortion get worse the higher is the leading breadth protection. [4]

All the aforementioned papers, except Cho and Shy (1993)[5], study economies of infinitely living agents in a continuous time framework. As it is well known from the classic (capital accumulation) growth literature, the outcomes of welfare and therefore policy analysis greatly differ between the two main conceptual vehicles for macroeconomic analysis: the

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3 In his analysis of endogenous innovation size, Li (2001) shows that looser lagging breadth protection results in larger innovation size and lower overall R&D effort.

4 In another recent paper Acemoglu and Akcigit (2009), propose optimal IPR policy which depends on the relative technological advantage of the industry leader.

5 Chou and Shy study optimal patent length in a variety expanding OLG economy. They show that old patents can crowd out new innovations in and OLG economy, where the young can invest in innovation or alternatively buy patent rights from the old.

Ramsey economy with infinitely living (and working) agents, and the OLG economy finitely living agent and life-cycle characteristics of labor supply and consumption. The present study provides a first attempt to fill this gap in the literature, namely the lack of a patent policy analysis for a quality-ladder OLG economy.

We develop a simple quality ladder model for the classic OLG economy (in the spirit of Dimond 1965). In such economy agents live for two periods: in the first period they work consume and save and in the second they consume their saving (while in the infinite horizon models labor supply is the same in all periods). We keep our model's characteristics as close as possible to the canonical reference by Grossman and Helpman (1991). Nevertheless, the inherited structural differences between the two frameworks have called for one significant modification in our model: we assume that the innovation process is certain, and therefore we exclude patent races within industries. This modification (to be explained in details in section 2) does not however affect our main qualitative results.

Our study starts with the analysis of stationary equilibrium in a decentralized economy under the benchmark patent policy: complete lagging breadth protection and no leading breath protection (as in Grossman and Helpman,1991), and a one period life patent length. Our patent-policy analysis focuses on optimal lagging and leading breadth, and minimal patentability requirement: We characterize the conditions for optimal R&D investment which maximizes welfare at the stationary equilibrium, which also define the conditions for welfare-improving minimal-patentability requirement. Then, we analyze the effects of changes in innovators marker power, through changes in patent breadth protection, on the stationary R&D investment and welfare. We find that these are determined by the inter-temporal elasticity of substitution (IES). Namely, we find that for plausible values of IES – that is lower than one[6] – weakening innovators market power by narrowing lagging breadth protection, counter – intuitively, has positive effect on R&D investment, but not necessarily on welfare. The crucial role of the IES in our patent-policy evaluation is unique to the OLG model, whereas in the analogous infinite horizon model it has no relevance for the steady state performance.

[6] See for example Hall (1998), Beaudry and Wincoop (1996), and Ogaki and Reinhart (1998) who estimate the IES to be smaller than one.

The remainder of the paper is organized as follows: section 2 presents the detailed model. Section three establishes the stationary equilibrium of the decentralized economy under the benchmark patent policy. Section 4 analyzes the effects of different patent breadth policies on the stationary growth rate and welfare. Section 5 discusses the results and section 6 concludes.

## 2. The model

We study a close overlapping-generations economy. In every period a unit mass of identical agents is being born to live two periods, and thus at every period two generations are alive – young and old. Agents are endowed with one unit of labor to be utilized (supplied) in the first period of life. We try to keep as close as possible to Grossman and Helpman (1991) reference model, by employing Cob-Douglas preferences, and linear technologies in labor. Nevertheless, the distinct demographic structure of the OLG framework has leaded us to assume a certain innovation process, whereas in the conventional R&D infinite-horizon models of continuous time, the innovation process is subject probabilistic success according to Poisson process. Consequently, in these models success probability is time invariant and the probability of joint success of two firms in one period is assumed to be zero. Adding the assumption of free entry to the R&D market opens the door for a patent race at the industry level that is easy to analyze. In the OLG framework however, time is discrete and each period expands over many years. Thus we cannot reasonably exclude the probability of joint success. Allowing for such however, greatly complicate calculation and exposition without altering our qualitative results. Because innovation process is certain, by assuming Bertrand competition at the products markets, we leave a room for only one innovator in each industry at each period. Hence in our model R&D intensity identifies with the quality steps[7]. In addition under the certain innovation process the arrival date of the new innovation is independent of R&D effort and, therefore, so is the effective patent-life of previous innovation. The relevance of this property to patent policy will be clarified in section 4. Finally, our benchmark policy is of complete lagging breadth, no leading breath or minimal patentability requirement, and a patent length protection of one period. The

[7] In the early model as in Grossman and Helpman (1991) quality steps are exogenous and R&D intensity is endogenous, later papers as Li (2001) and O'Dowoghue and Zwemuller (2004) consider both quality steps and R&D intensity as endogenous.

patent policy employed by Grossman and Helpman (1991) differs from ours only in patent length protection, which they assume to be infinite. In our model, extending patent length beyond one period gives room for intergenerational trade in patents: the young can buy patent from the old instead of innovating. From the practical stand point we like to abstract form this possibility to allow us focusing on patent breadth policy. From a positive stand point our assumption is much in accordance with the current patent policy implemented in the US and other industrialized economies where the patent length protection is given for about 15 years.

*Preferences*

Consumers born in period $t$ derive utility from consumption over their life span, according to the following utility function:

$$(1) \qquad U_t = U\left(C_t, C_{t+1}\right) = \frac{C_t^{1-\sigma}}{1-\sigma} + \rho \cdot \frac{{C_{t+1}}^{1-\sigma}}{1-\sigma}$$

Where $\frac{1}{\sigma}$ is the constant inter-temporal elasticity of substitution ( $\sigma > 0$ ), and $\rho$ is the subjective discount factor. $C_t$ is a consumption aggregator which aggregates consumption over a unit-continuum of different products, according to the Cobb- Douglas form: $C_t = \exp\left(\int_0^1 \ln q_{i,t} \cdot c_{i,t} di\right)$.

$q_{i,t}$ and $c_{i,t}$ stand for the utilized quantity and quality of product $i$ in period $t$. Consequently, within each period the demand for each good has unit elasticity with respect to both price and total expenditure: $c_{i,t} = \frac{E_t}{p_{t,i}}$. There is no capital or storage technology, and thus saving takes solely the form of investment in quality improvements. We denote saving rate of the young in period $t$ as $s_t$ .Under the assumed normalizations the saving rate will identify with saving level.

*Technology:*

Production and innovation take labor as a sole input. Production and innovation technologies are linear, symmetric across industries and constant over time. We normalize both production productivity and the wage to be one, and thus obtain a unit marginal production cost. Industry's R&D investment, denoted $R_i$ (in terms of labor units), improves over existing state-of-the-art product quality by a factor $\lambda_i$, with a one period lag. That is $q_{i,t+1} = \lambda_{i,t+1} \cdot q_{i,t}$, where:

(2) $\lambda_{i,t+1} = 1 + \phi \cdot R_{i,t}$

In our baseline analysis we assume full lagging breadth protection and no leading breadth protection as in Grossman and Helpman (1991) and Aghion and Howitt (1992). Under such patent breadth the official patent length is not effective, since one a quality improvement is being launched to the market the previous state of the art quality is being dropped out of the market, yet imposing a vertical competition which defines a limit price of the new latest improved quality product: $\frac{q_{i,t+1}}{p_{i,t+1}} = \frac{q_{i,t}}{mc}$. As the marginal production cost is normalized to one we obtain: $p_{i,t+1} = \lambda_{i,t+1}$. The profit in industry $i$ is given by:

(3) $\pi_{i,t+1} = \left(p_{i,t+1} - mc\right) c^{a}{}_{i,t+1} - R_{i,t} = \left(\frac{\lambda_{i,t+1} - 1}{\lambda_{i,t+1}}\right) E^{a}{}_{t+1} - R_{i,t}$

Where the upper script $a$ denotes aggregate variable, that is $c^{a}_{i,t+1}$ and $E^{a}_{i,t+1}$ are aggregate demand and aggregate expenditure in industry $i$ in period $t+1$, respectively, by both young and old generations.

## 3. Equilibrium

In equilibrium the labor market clears as young workers optimally allocate their labor between production and R&D investment. In addition the goods markets clear as output meets aggregate demand, where all products are being sold at their limit prices as were defined above. In equilibrium, R&D investment should yield equal return across sectors. As the (gross) return on R&D investment in industry $i$ is defined by the ratio of the industry's surplus over its R&D investment, we obtain:

$$(4)\ \left(1+r_t\right)=\frac{PS_{i,t}}{R^a{}_{i,t}}=\left(\frac{\lambda_{i,t}-1}{\lambda_{i,t}}\right)\frac{E^a{}_t}{R^a{}_{i,t}},\ \forall i$$

By the definition of the innovation technology in (2), $R^a{}_{i,t}=\frac{\lambda_{i,t}-1}{\phi}$, the equilibrium condition (4) can be written as:

$$(4a)\ \left(1+r_t\right)=\frac{\phi E^a{}_t}{\lambda_{i,t}},\ \forall i$$

As all industries share the same innovation technology condition (4a) is satisfied only when R&D investment across industries is equal. For each consumer born in period $t$ total expenditure in the first and second periods of life are given by $\left(1-s_t\right)$ and $s_t\left(1+r_{t+1}\right)$, Respectively. Thus, aggregate demand for each product in period $t+1$ is given by:

$$(5)\ c^a{}_{i,t+1}=\frac{E^a{}_{i,t+1}}{\lambda_{i,t+1}}=\frac{s_t\cdot(1+r_{t+1})+\left(1-s_{t+1}\right)}{\lambda_{i,t+1}}$$

Aggregate resources - uses constraint for each period implies that the saving of the young generation equals aggregate R&D investment:

$$(6)\ s_t=\int_0^1 R_{i,t}$$

By using definition (2) again, we obtain:

(6a) $s_t = \int_0^1 \frac{\lambda_{i,t+1} - 1}{\phi} \Rightarrow s_t + \frac{1}{\phi} = \frac{E_{t+1}}{(1+r_{t+1})}$

By substituting equation (4a) into (6a) we get the following expression for the returns on investment:

$$s_t + \frac{1}{\phi} = \frac{s_t \cdot (1+r_{t+1}) + (1-s_{t+1})}{(1+r_{t+1})}$$

(7)

$$\Rightarrow (1+r_{t+1}) = \phi \cdot (1-s_{t+1})$$

The return on saving is determined by R&D (relative) productivity and the demand of successive generation for the innovations of the current one. Based on symmetry across sectors we can write the indirect utility function as a function of saving rate, prices and qualities:

(8)

$$V(s) = \frac{\left(\exp\left(\int_0^1 \ln \frac{q_{i,t}}{\lambda_{i,t}} \cdot (1-s_t)\right)\right)^{1-\sigma}}{1-\sigma} + \rho \cdot \frac{\left(\exp\left(\int_0^1 \ln \frac{q_{i,t+1}}{\lambda_{i,t+1}} \cdot s_t \cdot (1+r_{t+1})\right)\right)^{1-\sigma}}{1-\sigma} =$$

$$= \frac{\left(\frac{q_{i,t}}{\lambda_{i,t}} \cdot (1-s_t)\right)^{1-\sigma}}{1-\sigma} + \rho \cdot \frac{\left(\frac{q_{i,t+1}}{\lambda_{i,t+1}} \cdot s_t \cdot (1+r_{t+1})\right)^{1-\sigma}}{1-\sigma}$$

Optimizing with respect to saving rate we obtain the following first order condition:

(9) $\left(\frac{s_t *}{1-s_t *}\right)^{\sigma} = \rho\left[(1+r_{t+1}) \cdot \lambda_t\right]^{1-\sigma}$

Substituting (7) into (9) we obtain:

(9a) $\left(\frac{s_t *}{1-s_t *}\right)^{\sigma} = \rho\left[\phi(1-s_{t+1})(\lambda_t)\right]^{1-\sigma}$.

Note that $\lambda_t = 1 + \phi \cdot s_{t-1}$. Focusing on the stationary equilibrium we impose $s_{t-1}^* = s_t^* = s_{t+1}^* \equiv s^*$, and $\lambda_t^* = \lambda_{t+1}^* \equiv \lambda^*$, and rewrite (9a) as:

$$\left(\frac{s^*}{1-s^*}\right)^{\sigma} = \rho \cdot \left[\phi \cdot (1-s^*) \cdot (\lambda_t)\right]^{1-\sigma}$$

Finally, after multiplying both sides of (10) by $\left(\frac{s^*}{1-s^*}\right)^{1-\sigma}$, we end with:

(10) $$\frac{s^*}{1-s^*} = \rho\left[(\lambda^* - 1) \cdot \lambda^*\right]^{1-\sigma}$$

Equation (10) defines the stationary equilibrium. Let us denote the right and left side of (10), $f(s^*)$ and $g(s^*)$, respectively. Clearly a stationary equilibrium exists at any intersection of $f$ and $g$. $f$ is a convex, monotonically increasing function of $s^*$, which goes from zero to infinity within the interval $[0,1]$. For $\sigma < 1$, $g$ is also a monotonically increasing function of $s^*$, within this domain, varying from zero to $\rho\left[\phi(1+\phi)\right]^{1-\sigma}$. Thus there is always a trivial stationary equilibrium with zero investment. At zero, the slope of $f'(s^*) = 0$, while $g'(s^*) \to \infty$, thus a non-trivial stationary equilibrium must exist. For $\sigma > 1$, $g$ is monotonically decreasing in $s^*$, from infinity to $\rho\left[\phi(1+\phi)\right]^{1-\sigma}$, and thus a unique positive stationary rate of saving exists. The stationary saving rate always increases with $\rho$. For $\sigma < 1$ the stationary saving rate increases with $\phi$, and vice versa. The stability of the stationary equilibrium is difficult to analyze since current optimal saving is affected by the saving decisions of both past and future generations. We could simplify the dynamic analysis by assuming that labor supply to production markets in inelastic and investment takes the form of final goods. In this case the stationary equilibrium is stable for every $\sigma > 1$, and for $\sigma < 1$ a sufficient condition for stability would be: $\frac{\sigma}{1-\sigma} > \phi$.

However such a simplification in of the dynamic analysis comes at the cost of departure from the analogous infinite horizon models. Thus, we will stick to our original formulation.

## 4. Patent Policy

In this section we compare the stationary rate of technological progress defined by optimal R&D with the social optimum, and study the marginal welfare effects of changes in minimum patentability requirement and patent breadth protection (compared with the benchmark policy). As it is well known, R&D-driven growth models are characterized by several externalities including the appropriability problem, efforts duplication (in the case of patent races which are absent from the present model), knowledge spillovers and the business stilling effect. In the presence of these externalities that are associated with the innovation process, one should not expect the decentralized economy to perform optimally (efficiently) under the base line patent policy, even with the infinitely living agents. Indeed, infinite horizon models conclude that under the base line patent policy the rate of technological progress (the quality steps) tends to be too low, while R&D intensity may be higher or lower than optimal (Grossman and Helpman, 1991, O'Dowoghue and Zwemuller, 2004). Yet it is left for us to identify and characterize the inefficiencies in the corresponding OLG agent's economy and their corrective policy measures.

### *4.1 Optimal technological progress and minimal patentability requirement*

First we like to identify the optimal R&D investment, which defines an optimal rate of technological progress, to be set by the social planner who maximizes the stationary lifetime utility of the representative agent. Denote the social planner set the rate of technological progress to be $\underline{\lambda}$. Note that whenever this progress rate is higher than the actual one, that is $\underline{\lambda} > \lambda^*$, the social planner's rate $\underline{\lambda}$ can be translated into an effective patentability requirement for minimal innovation size[8]. Under the regulated innovation size $\underline{\lambda}$ the price of new products becomes:

[8] Note also that under uncertain innovation process setting such a minimal patentability requirement as may also decrease the arrival rate of innovation, in case where success probability decreases with innovation size

$p_t = \underline{\lambda}$, the saving and interest rates are given by $s = \frac{\underline{\lambda}-1}{\phi}$ and $\phi - (\underline{\lambda}-1)$, respectively, and the corresponding indirect utility function becomes:

$$(11) \qquad V_t(\underline{\lambda}) = \left(\frac{1}{1-\sigma}\right)\left[\left(\frac{q_t \cdot \left(1-\frac{\underline{\lambda}-1}{\phi}\right)}{\underline{\lambda}}\right)^{1-\sigma} + \rho \cdot \left(q_t \cdot (\underline{\lambda}-1) \cdot \left(1-\frac{\underline{\lambda}-1}{\phi}\right)\right)^{1-\sigma}\right]$$

Differentiating (11) for $\underline{\lambda}$ we obtain the following expression:

$$(12) \qquad \rho \cdot \left(1-\frac{2(\underline{\lambda}-1)}{\phi}\right)(\underline{\lambda}-1)^{-\sigma} - \frac{\phi+1}{\phi\underline{\lambda}^2}\left(\frac{1}{\underline{\lambda}}\right)^{-\sigma}$$

If (12) is positive when evaluated at $\underline{\lambda} = \lambda^*$, the actual (stationary) rate of technological progress is lower relative to social optimum and thus marginal effective tightening of patentability requirement is welfare improving at the stationary equilibrium. Rearranging (12) we find it to be positive when:

$$(12a) \qquad \rho\left(\underline{\lambda}(\underline{\lambda}-1)\right)^{1-\sigma} > \frac{(\underline{\lambda}-1)(\phi+1)}{\left(\phi-2(\underline{\lambda}-1)\right)\underline{\lambda}}$$

Evaluating condition (21a) at $\underline{\lambda} = \lambda^*$ we recall that its left side equals $\frac{s^*}{1-s^*}$, and thus condition (12a) is satisfied only if:

$$(12b\ ) \qquad \frac{1}{1-s^*} > \frac{(\phi+1)}{(1-2s^*)(\phi s^*+1)}$$

**Proposition 1:** if $s^* > \frac{1}{2}$ $\left(s^* < \frac{1}{2}\right)$, the stationary rate of technological progress is too low (high) compared to social optimum. It happens when $\phi$ is high (low) enough relative to $\rho$ and $\sigma$. Therefore, in the case of $s^* > \frac{1}{2}$ tightening minimal patent requirement is welfare improving.

Proof:

Clearly, condition (12b) is satisfied whenever $s^* > \frac{1}{2}$, but it can be also, easily, verified that for $s^* < \frac{1}{2}$ the right side of (12b) is greater than the right side, as long as $\phi > 0$. According to (10) the stationary saving rate is greater than half if: $4 \cdot \rho^{\frac{1}{\sigma-1}} < \phi \cdot \left(\phi \frac{1}{2} + 1\right)$, that is if $\phi > \sqrt{1 + 8\rho^{\frac{1}{\sigma-1}}}$, and it is lower than half otherwise. Q.E.D.

Note that as $\rho$ approaches one and $\sigma$ is increasing well above one, this condition converges to $\phi > 3$. However when $\sigma$ approaches one from above and $\rho < 1$, sufficient condition for to much investment is $\phi > 1$, while for $\sigma, \rho < 1$ the minimal productivity requirement is greater than three. If we would assume the investment takes the form of final good (instead of labor), we would get that the stationary saving rate always exceeds the social optimum. The reason for this extreme result stems from the intergenerational nature of trade. Recall that under the monopolistic limit price the only gain form investment stems from selling to the young generation.

*4.2 Lagging breadth:*

Here we study uniform incomplete lagging patent protection. Such an incomplete protection allows the production and provision of inferior-quality variants of the latest innovation, which can be interpreted as limited imitation. The policy is defined by a maximal level of imitation: $\bar{\lambda} \leq \gamma \cdot \lambda_{t,i}$, where $0 < a < 1$. We assume that imitation is costless and so the availability of imitation intensifies the vertical competition and thus restricts innovators mark-up to be $\bar{\lambda}$, which is smaller than the size of their innovation.[9] Note that such a policy is equivalent to price regulation aimed to restrict innovator markup for a given innovation size, as for example is the case for pharmaceuticals in Europe and Canada. Under the policy $\bar{\lambda}$, the i[th] industry profit is given by:

$$(13) \qquad \pi_{i,t} = \left( \frac{\bar{\lambda} - 1}{\bar{\lambda}} \right) E_t^{\,a} - R_{i,t}$$

Accordingly, the "equal-returns" condition becomes:

$$(14) \qquad \left(1 + r_{t+1}\right) = \frac{PS_{i,t}}{R_{i,t}} = \frac{\phi\left(\bar{\lambda} - 1\right) E_t^{\,a}}{\bar{\lambda}\left(\lambda_{i,t+1} - 1\right)}$$

Applying the aggregate resources-uses constraint as we did in section 3, we derive the following, corresponding expression for the rate of return on R&D investment:

$$(15) \qquad (1 + r_{t+1}) = \frac{\left(\bar{\lambda} - 1\right) \cdot (1 - s_{t+1})}{s_t}$$

Relying once again on symmetry, we modify consumer's indirect utility function as follows:

$$(16) \qquad V_t\left(s_t\right) = \frac{1}{1-\sigma} \cdot \left\{ \left[ \frac{q_t\left(1 - s_t\right)}{\bar{\lambda}} \right]^{1-\sigma} + \rho \left[ \frac{q_t \cdot \lambda_{t+1} \cdot s_t \cdot (1 + r_{t+1})}{\bar{\lambda}} \right]^{1-\sigma} \right\}$$

Optimizing (16) for saving rate and solving for the stationary equilibrium, we obtain:

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[9] Obviously, the assumed a cost-free imitation is not a realistic assumption Nevertheless this is the simplest conventional way to model competition intensifying through imitation.

(17) $\left(\frac{s*}{1-s*}\right)=\rho\cdot\left(\lambda*\cdot\left(\bar{\lambda}-1\right)\right)^{1-\sigma}$

Equation (17) implies that the effect of on the stationary saving rate and thereby growth is determined by inter-temporal elasticity if substitution.

**Proposition 2**: When IES is smaller (greater) than one the stationary saving rate increases (decreases) with marginal loosening of lagging-breadth protection. That is when $\sigma>1(\sigma<1)$, $\frac{ds*}{d\bar{\lambda}}<0\,(\frac{ds*}{d\bar{\lambda}}>0)$.

Proof:

Because the elasticity of inter-temporal substitution is given by $\frac{1}{\sigma}$, For $\sigma>1(\sigma<1)$ IES is smaller (greater) than one. In this case the right side of (17) has a negative (positive) power and thus it increases (decreases) with the marginal decrease of $\bar{\lambda}$ .Q.E.D.

Proposition 2 implies the following non intuitive results: when IES is smaller than one loosening leading breadth protection spurs R&D investment and therefore technological progress. Obviously, this effect stems from consumers saving decisions determined by the IES: a lower (regulated) markup results in a lower interest rate. However, when IES is smaller than one the lower interest rate results in a higher optimal saving.

We turn now to evaluate the effect of loosening lagging breadth protection on welfare. Rearranging equation (17), evaluated at the steady state saving level, we derive the following indirect utility function which is the measure for the stationary welfare of the representative consumer:

(18) $V_t*\left(\bar{\lambda}\right)=\left(\frac{q_t}{\bar{\lambda}}\right)^{1-\sigma}\cdot\left(1-s*\left(\bar{\lambda}\right)\right)^{-\sigma}$

Differentiating (18) for $\bar{\lambda}$ we obtain the following first order condition for optimal policy:

(19) $$\frac{1-\sigma}{\sigma}\cdot(1-s)=\frac{ds^*}{d\bar{\lambda}}\bar{\lambda}^*$$

Where $\bar{\lambda}^*$ stands for the policy which maximizes steady state welfare. While it is difficult to derive an explicit solution for this implicit function, we are able to define some useful conditions under which incomplete lagging breath is, qualitatively, welfare enhancing that is when:

(19a) $$\frac{1-\sigma}{\sigma}\cdot(1-s)-\frac{ds^*}{d\bar{\lambda}}\bar{\lambda}^*<0$$

In order to do elaborate condition (19a) we apply the implicit function theorem and derive the following explicit expression for $\frac{ds^*}{d\bar{\lambda}}$ :

(20) $$\frac{ds^*}{d\bar{\lambda}}=\frac{(1-s)(1-\sigma)\cdot\frac{\bar{\lambda}}{\phi}}{1-(1-s)\cdot(1-\sigma)\cdot\frac{\phi\cdot s}{\lambda^*}}$$

Substituting (20) into (19a) we obtain:

(21) $$\frac{(1-\sigma)\cdot(1-s^*)}{\sigma}\left[1-\frac{\frac{\sigma\cdot\bar{\lambda}}{\phi}}{1-(1-s)\cdot(1-\sigma)\cdot\frac{\lambda^*-1}{\lambda^*}}\right]<0$$

In order for condition (21) to be satisfied the term in the brackets should be negative for $\sigma<1$, and positive for $\sigma>1$. In order for this term to be negative or negative, the following conditions should hold, respectively:

(21a) $$1-(1-s)\cdot(1-\sigma)\cdot\frac{\phi\cdot s^*}{\lambda^*}-\frac{\sigma\cdot\bar{\lambda}}{\phi}<0$$

(21b) $1-(1-s)\cdot(1-\sigma)\cdot\frac{\phi\cdot s^*}{\lambda^*}-\frac{\sigma\cdot\bar{\lambda}}{\phi}>0$

**Proposition 3**: For extreme values of $\sigma$ marginal lagging breadth loosening is not efficient.

Proof:

Evaluation of (19a) at $\sigma\to 0$ results in contradiction:

$$1<(1-s)\cdot\left(\frac{\lambda^*-1}{\lambda^*}\right)$$

When $\sigma$ is high enough, according to (10), optimal saving rate declines and approaches zero, therefore, condition (9b) is violated:

$$\frac{1}{\sigma}+\frac{(\sigma-1)}{\sigma}\frac{1}{(1+\phi\cdot s^*)^2}>\frac{1}{\phi^2(1-s^*)s^*}$$

Q.E.D.

**Proposition 4**: When $\sigma$ approaches one from below (above) and $1+\rho>\phi$ ($1+\rho<\phi$), marginal loosening of lagging breadth protection is welfare improving.

Proof:

When $\sigma$ approaches 1 from both above and below the optimal saving rate approaches $\frac{\rho}{1+\rho}$, and thus the term in the brackets of (21) approaches: $\frac{\phi-(1+\rho)}{(1+\rho)\phi}$. This term is negative (positive) when $1+\rho$ is greater (smaller) than $\phi$. Q.E.D.

While proposition s 3 and 4 are confined to specific limit values of the elasticity of substitution, simulations results show that proposition 3 holds for a wide range of values for $\sigma$, with a corresponding range of values for $\rho$ and $\phi$.

*4.3 Leading Breadth (Incomplete)*

Leading breath protection extends innovators rights over future potential quality-improvements, Hence future innovators will not be allowed to produce (and sell) their product without being licensed by previous ones and paying royalties in return. The patent breadth protection can define a dichotomist quality improvement below which the new innovation is subject to complete licensing by the previous innovator, and above which it is patentable with no royalties to the previous innovator. Then, in the case of licensing there is a vertical collusion between two successive innovators. The effect of such collusion on the market outcomes depends on other dimensions of the patent policy, namely patent length and lagging breadth. If patent length protection is limited to one period – which means there is no lagging breadth protection after one period - such vertical collusion does not affect innovators market power and markup as they face the same vertical competition with imitators of previous innovation anyway. However, in case the patent length is set for two periods, for instance, and lagging breath protection is complete the stationary mark is squared because every two following innovators collude and the face no vertical competition. For detailed discussion on the implementations of leading breadth protection see O'Dowoghue and Zwemuller (2004). While the higher markup and extended patent right work to increase the profitability of each innovation the delay in return caused by the requirement to pay previous innovators and getting paid by future innovators erodes profitability. This negative effect, known as the back-loading effect, is discussed in details by Chu (2009). In the OLG framework the royalties' payment must be paid by the young (to the old) at once. Under such royalties' payment the available resources at the first period of life decrease, while the available resources in the second period increase, and thus their combined effect on optimal saving is negative. This negative effect is does not exists in the infinite horizon model where back-loading works only to decrease the present value of innovations.

## 5. Discussion

TO BE ADDED

## 6. Conclusions

TO BE ADDED

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